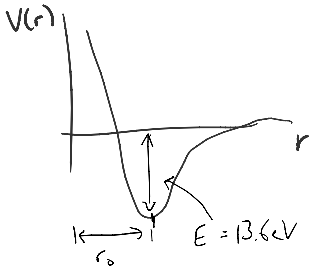
Binding energy



$V(r_\infty) - V(r_0) = 13.6\,\text{eV}$

System of H is
$m_H \sim (p + e + E_b)$

Analogy
$m_p \sim m_u + m_u + m_d + E_b$
proton is composite
Can separate constituents to be infinite far away
Bound system energy is again the diff. in energy from well-separated constituents to bound state.
$E_b = (m_p - 2m_u - m_d) \sim$ most of m_p

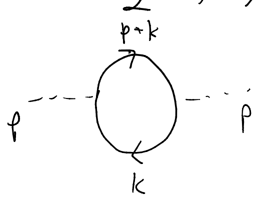
Special topics:

Hierarchy problem

Ref. Collins, "Renormalization" Section 8.2.

Calculate 1-loop corr. to scalar mass.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 + \lambda \phi \bar\psi \psi + \bar\psi (i\partial\!\!\!/ - M)\psi$$



$$i\Sigma_2(p) = (i\lambda)^2 \int \frac{d^4k}{(2\pi)^4} \cdot \frac{\mathrm{Tr}\left[(p\!\!\!/ + k\!\!\!/ + M)(k\!\!\!/ + M)\right]}{((p+k)^2 - M^2)(k^2 - M^2)}$$

$$\mathrm{Tr}[\ldots] = 4(k^2 + p\cdot k + M^2)$$

Feynman params:

$$= -4\lambda^2 \int \frac{d^4k}{(2\pi)^4} \cdot \int_0^1 dx \, \frac{k^2 + p\cdot k + M^2}{(k^2 + 2(p\cdot k)(1-x) + p^2(1-x) - M^2)^2}$$

$$(\tilde k - p(1-x))^2 + p\cdot(\tilde k - p(1-x)) + M^2$$
$$= \tilde k^2 + p^2(1-x)^2 - p^2(1-x) + M^2$$

Shift $\tilde k \equiv k + p(1-x)$; drop linear $\tilde k$ in numerator

$$= -4\lambda^2 \int \frac{d^4\tilde k}{(2\pi)^4} \cdot \int_0^1 dx \, \frac{\tilde k^2 + p^2(1-x)^2 - p^2(1-x) + M^2}{(\tilde k^2 + p^2(1-x) - M^2 - p^2(1-x)^2)^2}$$

$$= -4\lambda^2 \int \frac{d^4\tilde k}{(2\pi)^4} \int_0^1 dx \, \frac{\tilde k^2 + p^2[(1-x)^2 - (1-x)] + M^2}{(\tilde k^2 \quad \Delta)^2}$$

$$= -4\lambda^2 \int \frac{d^4\tilde{k}}{(2\pi)^4} \int_0^1 dx \; \frac{\tilde{k}^2 + p^2[(1-x)^2 - (1-x)] + M^2}{(\tilde{k}^2-\Delta)^2}$$

$$\Delta \equiv -p^2\left(1-x-(1-x)^2\right) + M^2$$
$$= -p^2\left(\cancel{1}\cancel{-x}\cancel{-1}+\cancel{2}x-x^2\right)+M^2$$
$$= -p^2(x)(1-x)+M^2$$

$$= -4\lambda^2 \int \frac{d^4\tilde{k}}{(2\pi)^4} \int_0^1 dx \; \frac{\tilde{k}^2-\Delta}{(\tilde{k}^2-\Delta)^2} + \frac{\Delta + p^2(1-x)(1-x-1)+M^2}{(k^2-\Delta)^2}$$

$$\Delta + p^2(1-x)(-x) + M^2$$
$$= 2M^2 + p^2(1-x)(-2x)$$
$$= 2M^2 - 2p^2(x)(1-x)$$
$$= 2\Delta$$
$$= -4\lambda^2 \int \frac{d^4\tilde{k}}{(2\pi)^4} \int_0^1 dx \; \frac{1}{\tilde{k}^2-\Delta} + \frac{2\Delta}{(k^2-\Delta)^2}$$

Using dim. reg.

$$= \frac{4\lambda^2}{(4\pi)^{d/2}} (d-1)\,\Gamma\left(1-\frac{d}{2}\right) \mu^{4-d} \int_0^1 dx \left(M^2 - x(1-x)p^2\right)^{\frac{d}{2}-1}$$

Pole @ $d=2$ [quad. div.].

Use $d=4-\epsilon$

$$= \frac{-\lambda^2}{4\pi^2}\left(\frac{6M^2}{\epsilon} - \frac{p^2}{\epsilon} + M^2 - \frac{1}{6}p^2 + \int_0^1 dx\left(3p^2x(1-x)-3M^2\right) \cdot \log \frac{\Delta}{4\pi\mu^2 e^{-\gamma}}\right)$$

$$= -4\lambda^2 \int \frac{d^4\tilde{k}}{(2\pi)^4} \int_0^1 dx \left(\frac{1}{\tilde{k}^2-\Delta} + \frac{2\Delta}{(k^2-\Delta)^2}\right)$$

$$\Delta = -p^2x(1-x) + M^2$$

$$\int \frac{d^4\tilde{k}}{(2\pi)^4} \frac{1}{\tilde{k}^2-\Delta} = \frac{(-1)}{(4\pi)^{d/2}} i \cdot \frac{\Gamma\left(1-\frac{d}{2}\right)}{1} \left(\frac{1}{\Delta}\right)^{1-d/2}$$

$$\int \frac{d^4\tilde{k}}{(2\pi)^4} \frac{2\Delta}{(\tilde{k}^2-\Delta)^2} = \frac{1}{(4\pi)^{d/2}} i \cdot \frac{\Gamma(2-d/2)}{1} \cdot \left(\frac{1}{\Delta}\right)^{2-d/2} \cdot 2\Delta$$

$$= \frac{-4\lambda^2}{(4\pi)^{d/2}}\, i \int_0^1 dx \left(-\frac{\Gamma(1-d/2)}{\Delta^{1-d/2}} + 2\Delta \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} \right)$$

$$= \frac{-4\lambda^2}{(4\pi)^{d/2}}\, i \int_0^1 dx \left(-\frac{\Gamma(1-d/2)}{\Delta^{1-d/2}} + 2\Delta \frac{\Gamma(2-d/2)}{\Delta^{2-d/2}} \right)$$

$$= \frac{-4\lambda^2}{(4\pi)^{d/2}}\, i \int_0^1 dx \left(\frac{1}{\Delta^{1-d/2}} \right)\left(-\Gamma(1-d/2) + 2\cdot(1-d/2)\,\Gamma(1-d/2) \right)$$

$$= \frac{-4\lambda^2}{(4\pi)^{d/2}}\, i \int_0^1 dx \; \frac{\Gamma(1-d/2)}{\Delta^{1-d/2}} \left(-1 + 2 - d \right)$$

$$= \frac{+4\lambda^2 i}{(4\pi)^{d/2}} \cdot (d-1) \int_0^1 dx \; \Gamma(1-d/2) \left(M^2 - p^2 x(1-x) \right)^{\frac{d}{2}-1} \quad \checkmark$$

$$= \frac{4\lambda^2 i\,(d-1)}{(4\pi)^{d/2}}\, \Gamma\left(1-\frac{d}{2}\right) \mu^{4-d} \int_0^1 dx \left(M^2 - x(1-x)p^2 \right)^{\frac{d}{2}-1}$$

$$\Gamma\left(1 - \frac{d}{2}\right)$$

For $x = -n$, $\Gamma(x) = \frac{(-1)^n}{n!}\left(\frac{1}{x+n} - \gamma + 1 + \frac{1}{n} + O(x+n) \right)$

...

$$\Sigma_2(p) = \frac{-\lambda^2}{4\pi^2}\left(\frac{6M^2}{\epsilon} - \frac{p^2}{\epsilon} + M^2 - \frac{1}{6}p^2 + \int_0^1 dx \left(3p^2 x(1-x) - 3M^2\right) \cdot \log \frac{\Delta}{4\pi\mu^2 e^{-\gamma}} \right)$$

$$\frac{d\Sigma}{dp^2} = \frac{-\lambda^2}{4\pi^2}\left(\frac{-1}{\epsilon} - \frac{1}{6} + \int_0^1 dx\, 3x(1-x) \cdot \log \frac{\Delta}{4\pi\mu^2 e^{-\gamma}} + \int_0^1 dx \; \frac{\left(3p^2 x(1-x) - 3M^2\right)\cancel{\left(4\pi\mu^2 e^{-\gamma}\right)}}{\Delta \;\cancel{4\pi\mu^2 e^{-\gamma}}} \cdot \frac{d\Delta}{dp^2} \right)$$

$$\frac{d\Delta}{dp^2} = -x(1-x)$$

$$\frac{d\Sigma}{dp^2} = \frac{-\lambda^2}{4\pi^2}\left(\frac{-1}{\epsilon} - \frac{1}{6} + \int_0^1 dx \left(\cdots \right) \right)$$

We calculate Σ_2 for scalar field.

Both Σ_2 and $\frac{d\Sigma_2}{dp^2}$ have divergences.

Σ_2 $\downarrow$ Removed by mass counterterm. $\frac{d\Sigma_2}{dp^2}$ Removed by field strength CT.

$$— \otimes - - - = i\left(p^2 \delta_\phi - (\delta_m + \delta_\phi)\, m_\chi^2\right)$$

Then, for $iG(p^2) = \frac{1}{p^2 - m^2 + \Sigma(p^2)}$

$$\Sigma(p^2) = \Sigma_2(p^2) + p^2\delta_\phi - (\delta_m + \delta_\phi) m_R^2$$

$$\& \quad \Sigma(m_p^2) = \frac{d}{dp^2}\Sigma(m_p^2) = 0 \quad @ \; m_p = m_R$$

$$\Rightarrow \delta_m = \frac{1}{m_p^2}\Sigma_2(m_p^2)$$

$$\delta_\phi = -\frac{d}{dp^2}\Sigma_2(p^2)\Big|_{p^2 = m_p^2}$$

Calculate

$$\delta_m = \frac{-\lambda^2}{(4\pi^2)}\left[\left(\frac{6M^2}{m_p^2} - 1\right)\frac{1}{\epsilon} + \left(\frac{1}{2} - \frac{3M^2}{m_p^2}\right)\ln\frac{M^2}{\tilde{\mu}^2} + \frac{M^2}{m_p^2} + \frac{1}{3} - \frac{m_p^2}{20M^2} + \mathcal{O}\left(\frac{m_p^4}{M^4}\right)\right]$$

$$\delta_\phi = \frac{-\lambda^2}{4\pi^2}\left[\frac{1}{\epsilon} - \frac{1}{2}\ln\frac{M^2}{\tilde{\mu}^2} - \frac{1}{3} + \frac{m_p^2}{10M^2} + \mathcal{O}\left(\frac{m_p^4}{M^4}\right)\right]$$

$$\Sigma(p^2) = \frac{\lambda^2}{4\pi^2}\left[\frac{(p^2 - m_p^2)^2}{20M^2} + \mathcal{O}\left(\frac{m_p^6}{M^4}\right)\right]$$

For $\overline{MS}$, no finite parts,

$$\delta_m = \frac{-\lambda^2}{4\pi^2\epsilon}\left(\frac{6M^2}{m_p^2} - 1\right)$$

$$\delta_\phi = \frac{-\lambda^2}{4\pi^2\epsilon}$$

Finite corrections $\quad m_p^2 - m_{\overline{MS}}^2(\mu) = \frac{\lambda^2}{24\pi^2}(6M^2 - m_p^2) - \frac{3\lambda^2}{4\pi^2}\int_0^1 dx\, \Delta \ln\frac{\Delta}{\mu^2}$

find that difference is proportional to M^2.

Recall M is mass of fermion in loop.

M can be very large, but correction does <u>not</u> <u>decouple</u>.

Need to know about energy scales $M^2 \gg m_p^2$ to understand at m_p^2.

Heart of hierarchy problem: Definition of counterterms encodes true $\frac{1}{\epsilon}$ div. (UV div.) and finite non-decoupling.

Heart of hierarchy problem: Definition of counterterms encodes true $\frac{1}{\epsilon}$ div. (UV div.) and finite, non-decoupling UV sensitivity.

For extensions of model, M scale is physical scale. Always new d.o.f. at $M_1, M_2, M_3 \Rightarrow$ change how you have to define CT @ low scale m_p.

Finally, if m_h is calculable, then this UV sensitivity is inescapable $\Leftrightarrow$ fine tuning.

$$m_W = \frac{gv}{2}, \qquad m_Z = \frac{\sqrt{g^2+g'^2}}{2}\, v$$

$$m_h = \lambda v^2$$

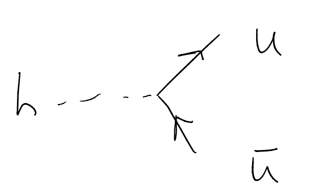
$$G_F = \frac{1}{2\sqrt{2}v^2}$$

SUSY: No independent λ. Instead λ is replaced by $f(g^2, g'^2)$

$$g^2 |H_u|^4 + g'^2 |H_d|^4$$

Homework 6:

6-1. A.)

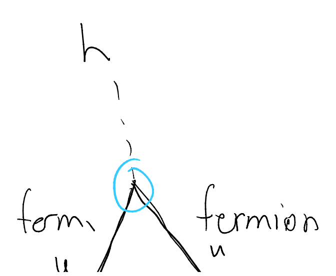


$$\mathcal{L} = -\frac{y_u}{\sqrt{2}} h \bar{u} u$$

$$-i\frac{y_u}{\sqrt{2}}$$

B.)

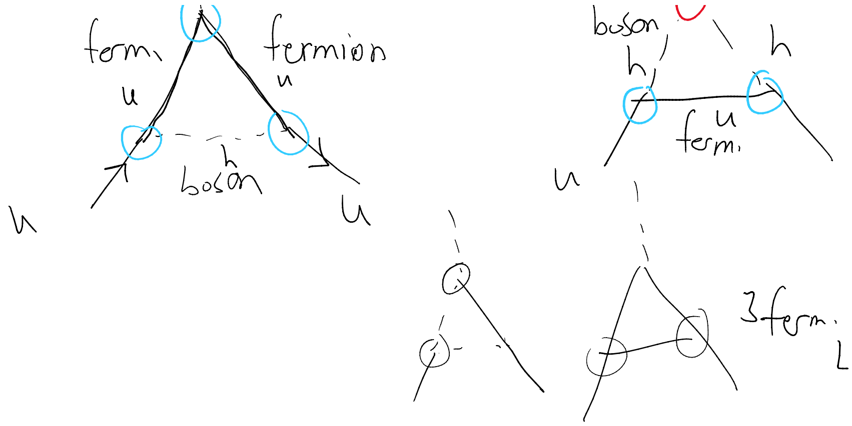
C.)



$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$\mathcal{L} = \lambda v h^3$$

C.)



3 ferm. not allowed
Lorentz breaking

$\mathcal{L} = \lambda v h^3$

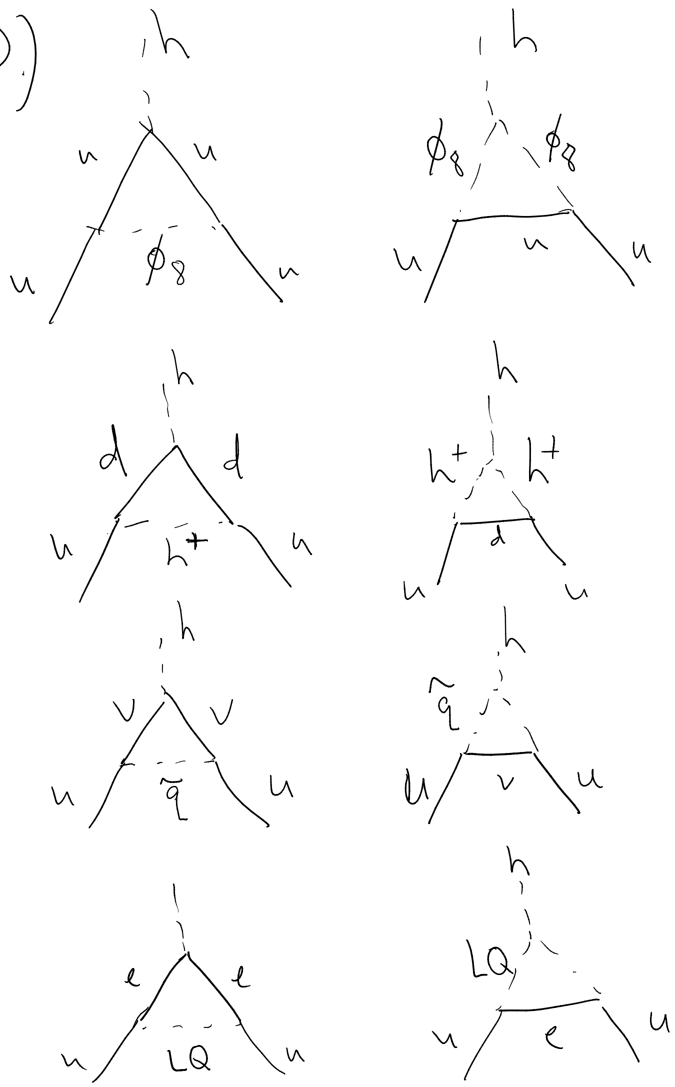
from $\lambda|H|^4$

$= \frac{\lambda}{4}\begin{pmatrix}0\\ v+h\end{pmatrix}^4$

$= \frac{\lambda}{4}\cdot 4vh^3$

$= \lambda v h^3$

D.)



$SU(3) \times U(1)_{em}$

$\phi_8 \sim (8, 0)$

$h^+ \sim (1, 1)$

$(\text{or } h_8^+ \sim (8, 1))$

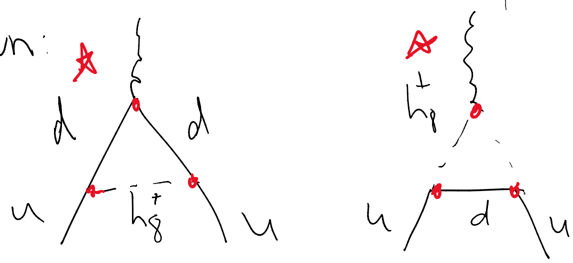
$\tilde{q} \sim (3, 2/3)$

$LQ \sim (3, \frac{5}{3})$

E.)

down:

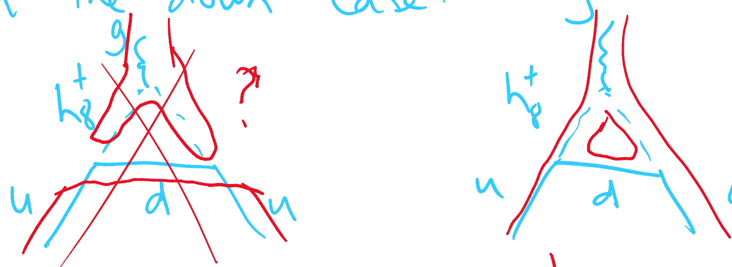
down:



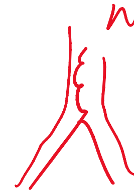
Will all have $g_\mu^a t^a$ $\bar{u}\gamma^\mu u$ structure

electron:

In the down case:



using double line notation



Same as tree level.

Triplets (fundamental) : $\xrightarrow{\text{color}}$

Anti fundamental $\longleftarrow$

Adjoints $\overrightarrow{\underleftarrow{\quad}}$

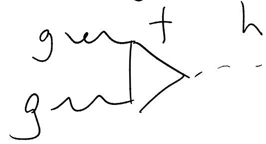
Tracks color flow

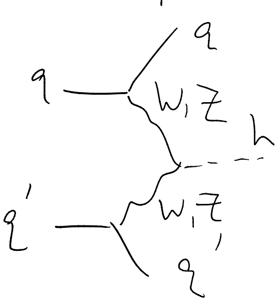
6-2.

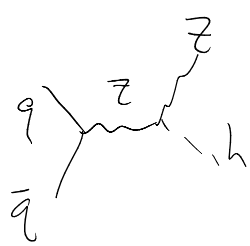
A. $h \to ZZ^* \to 4\ell$ (not $h \to WW^* \to \ell\nu\ell\nu$)

$h \to \gamma\gamma$

B. $gg \to h$ VBF VBA



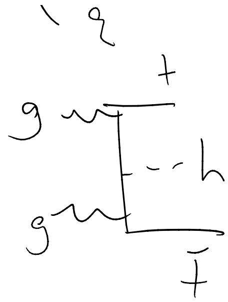




$t\bar{t}h$

t

$t\bar{t}h$



C. $m_H = 125.1$ GeV

$m_Z = 91.1876$ GeV

$m_W = 80.379$ GeV

$m_t = 172.76$ GeV

$m_b = 4.18$ GeV ($\overline{MS}$)

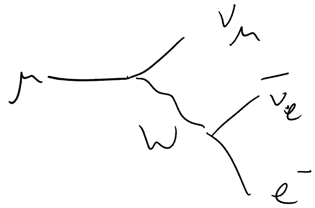
D. $m_e = 511$ keV

$m_\mu = 105.66$ MeV

$m_\tau = 1.77686$ GeV

E. $Br(t \to bW) \approx 100\%$

F. μ lifetime



G. QCD confinement

H. BaBar + Belle Υ_{4S} + Υ_{5S}

I. Cabibbo angle

$\sin\theta_{12} \sim 0.2265$

$$V_{CKM} \sim \begin{pmatrix} 0.97 & 0.22 & \sim 0 \\ 0.22 & 0.97 & \sim 0 \\ \sim 0 & \sim 0 & 0.99 \end{pmatrix}$$