Lecture 16, July 1, 2020

Wednesday, July 1, 2020 2:04 PM

Last time: Gauge fixing in spont. broken theories

Goldstone's theorem

Match between Goldstones + gauge dofs. in path integral for Abelian theory; R_ξ gauge

Today: Non-Abelian gauge fixing + Goldstones + ghosts

Higgs mechanism in SM, W + Z boson masses, fermion masses + CKM matrix

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When we analyze non-Abelian theory, follow same analysis as Abelian theory.

field $\delta\phi_i = -\alpha^a(x) \underbrace{T^a_{ij}\phi_j}_{\Delta^a \text{ from before}}$ ϕ_i is multiplet of real-valued fields

$D_\mu \phi_i = \partial_\mu \phi_i - i g A^a_\mu T^a_{ij} \phi_j$

$\phi_i = v_i + \chi_i(x)$ Goldstones: field fluctuations along directions spanned by $T^a_{ij} v_j$.

Physical scalars: orthogonal dirs.

Convenient notation: $F^a_i = T^a_{ij} v_j$

↳ Matrix is not necc. square.

$$Z = \int \mathcal{D}A\mathcal{D}\chi\, e^{i\int \mathcal{L}[A,\chi]}$$

Fadeev-Popov procedure

$$= C' \int \mathcal{D}A\mathcal{D}\chi \exp\left[i\int d^4x\, \mathcal{L}[A,\chi] - \tfrac{1}{2}(G)^2\right] \det\left(\frac{\delta G}{\delta \alpha}\right)$$

Gauge fixing fn.

$$G^a = \frac{1}{\sqrt{\xi}}\left(\partial_\mu A^{a\mu} - \xi g F^a_i \chi_i\right) \qquad \left[\text{cf. } \frac{1}{\sqrt{\xi}}\left(\partial_\mu A^\mu - \xi e v \varphi\right) \text{ in } U(1) \text{ case}\right]$$

Mixing

a_μ ~~~×- - - χ_i

vs.

W^+, W^-, ϕ^+, ϕ^-

Interaction

See left

Evaluate: quadratic part of $-\frac{1}{2}G^2$ is

mixing term between Goldstone χ_i + Gauge field

$$= \frac{1}{2} A^a_\mu \left(\frac{1}{\xi}\partial^\mu\partial^\nu\right) A^a_\nu + \boxed{g\,\partial_\mu A^{a\mu} F^a_i \chi_i} - \frac{1}{2}\xi g^2 [F^a_i \chi_i]^2$$

$$= -\frac{1}{2} A^a_\mu \left[\left(-g^{\mu\nu}\partial^2 + \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right)\delta^{ab} - g^2 F^a_i F^b_i g^{\mu\nu}\right] A^b_\nu$$

$$+ \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}\xi g^2 F^a_i F^a_j \chi_i \chi_j$$

↳ mass term for gauge fields

no mixing term, eliminated by EOM

$$\text{Ghost} \quad \det\left(\frac{\partial G^a}{\partial \alpha^b}\right) = \det\left[\frac{1}{\sqrt{\xi}}\left(\frac{1}{g}(\partial_\mu D^\mu)^{ab} + \xi g (T^a v)\cdot T^b (v+\chi)\right)\right]$$

$$\mathcal{L}_{ghost} = \bar{c}^a\left[-(\partial_\mu D^\mu)^{ab} - \xi\, g^2 T^a\phi_0 \cdot T^b(\phi_0 + \chi)\right]c^b$$

In Electroweak theory, these ξ-dependent propagators are

$\mu \quad \nu$ (k): $$\frac{-i}{k^2-M^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2-\xi M^2}(1-\xi)\right)$$

$M^2 = \underset{W^{+/-}}{m_W^2}\,;\ \underset{Z}{m_Z^2}\,;\ \underset{\text{photon}}{m_A^2} = 0$

$$\frac{i}{k^2-\xi M^2}$$ 3 Goldstones

$M^2 = \underset{\varphi^{+/-}}{m_W^2}\,,\ \underset{\varphi^0}{m_Z^2}$

$$\frac{i}{k^2-\xi M^2}$$ 4 ghosts

$M^2 = m_W^2,\ m_Z^2,\ m_A^2 = 0$

R_ξ gauge appropriate for spont. broken non-Abelian gauge theory.

$\mathcal{L} \supset |D_\mu H|^2$

$H = \frac{1}{\sqrt{2}}\begin{pmatrix} i\phi^+ \\ v + h + i\phi^0 \end{pmatrix}$

$D_\mu \supset W_\mu^+,\ W_\mu^-$

Then $\mathcal{L} \supset \frac{g^2}{2}\, W_\mu^+\phi^+\, W^{-\mu}\phi^-$ from quartic piece.

R_ξ choices: Can self-consistently choose any ξ to get gauge-invariant result.

All physical quantities are ξ-independent.

$\mu \quad \nu$: $$\frac{-i}{k^2-M^2}\left(g^{\mu\nu} - \frac{(1-\xi)k^\mu k^\nu}{k^2-\xi M^2}\right)$$

$$\frac{i}{k^2-\xi M^2}$$ goldstones

$$\frac{i}{k^2-\xi M^2}$$ ghosts

Note in $\xi \to \infty$, no Goldstones or Ghosts.

$\mu \quad \nu$: $$\frac{-i}{k^2-M^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{M^2}\right)$$

But in $\xi \to 1$ ...

$\mu \sim\sim \nu \quad \frac{i}{k^2-M^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{M^2}\right)$

But in $\xi \to 1$, we have

$\mu \sim\sim \nu \quad \frac{-i}{k^2-M^2}(g^{\mu\nu})$

difference is manifest as new d.o.f. in Goldstones

$- - - - \quad \frac{i}{k^2-M^2}$

$\cdots\cdots \quad \frac{i}{k^2-M^2}$

Back to SM + Higgs mechanism + spont. breaking of Electroweak symmetry.

$H \sim (1, 2, 1/2)$

$\mathcal{L}_H \supset |D_\mu H|^2 - \mu^2 |H|^2 - \lambda |H|^4, \quad \mu^2 < 0$

$D_\mu H = \partial_\mu H - ig\tau^i W^i_\mu H - \frac{ig'}{2} B_\mu H$

Study $\langle H \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$ [Recall $\tau^i = \frac{\sigma^i}{2}$, generators of $SU(2)$]

$v^2 = -\frac{\mu^2}{\lambda}$

$D_\mu H \supset -ig\frac{\tau^i}{\sqrt{2}} W^i_\mu \begin{pmatrix} 0 \\ v \end{pmatrix} - \frac{ig'}{2\sqrt{2}} B_\mu \begin{pmatrix} 0 \\ v \end{pmatrix}$

$\tau^i W^i_\mu = \frac{1}{2}\left(\sigma^1 W^1_\mu + \sigma^2 W^2_\mu + \sigma^3 W^3_\mu\right)$

$= \frac{1}{2}\begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{pmatrix}$ (e.g. matrix valued field)

$\sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$D_\mu H = \begin{pmatrix} \frac{1}{2\sqrt{2}}(-ig)(W^1_\mu - iW^2_\mu)v \\ \frac{1}{2\sqrt{2}}(-ig)(-W^3_\mu)v - \frac{ig'}{2\sqrt{2}} B_\mu v \end{pmatrix}$

$|D_\mu H|^2 = \frac{1}{8}\left(g^2 v^2 (W^1_\mu - iW^2_\mu)(W^1_\mu + iW^2_\mu)\right)$
$+ \frac{1}{8} v^2 (-gW^3_\mu + g'B_\mu)^2 \Rightarrow v^2 \begin{pmatrix} g^2 & gg' \\ gg' & g'^2 \end{pmatrix}\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix}$

one lin. combination of W^3 + B that is massive.

During break, perform next step:
find mass eigenstates + masses.

Return @ 3:10 pm.

We identify:

We identify:

Need complex vector field:

$m_X^2 X^+_\mu X^{-\mu}$ or

real vector field:

$\frac{m_Y^2}{2} Y_\mu Y^\mu$

$\Rightarrow$ Identify $\frac{1}{\sqrt{2}}(W_\mu^1 - iW_\mu^2) = W^+$

$\frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2) = W^-$

& need to have kinetic terms canonically normalized

$\left[\text{e.g. } \frac{-1}{4} \underbrace{W^a_{\mu\nu} W^{a\mu\nu}} \rightarrow \frac{-1}{4} 2W^+_{\mu\nu} W^{-\mu\nu} \right]$

$m_W^2 = \frac{g^2 v^2}{4} \Rightarrow m_W = \frac{gv}{2}$; $m_W \simeq 80.4\,\text{GeV}$

$v \simeq 246\,\text{GeV}$

$g \simeq 0.65$

Identify: $Z_\mu = \frac{(-g W^3_\mu + g' B_\mu)}{\sqrt{g^2 + g'^2}}$

$\Rightarrow m_Z^2 = \frac{(g^2 + g'^2)}{4} v^2 \Rightarrow m_Z = (\sqrt{g^2 + g'^2}) \frac{v}{2}$

$m_Z \simeq 91.2\,\text{GeV}$

Last dof: $A_\mu = \frac{(g' W^3_\mu + g B_\mu)}{\sqrt{g^2 + g'^2}}$ $\leftarrow$ lin. comb. orthogonal to Z

No mass. $\Rightarrow$ Photon field.

$\mathcal{L} \supset -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$ (W^+, W^-)

$+ |D_\mu H|^2$

Isolating terms quadratic in gauge fields

$-\frac{1}{4} W^3_{\mu\nu} W^{3\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$

$m_Z^2 Z_\mu Z^\mu \left[+ \frac{1}{8} v^2 (-g W^3_\mu + g' B_\mu)^2 \right.$

$m_Z^2 Z_\mu Z^\mu \Big| + \frac{1}{8} v^2 \left(-g W^3_\mu + g' B_\mu \right)^2$

$$\mathcal{L} \supset -\frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

$$-\frac{1}{4} g'^2 W^3_{\mu\nu} W^{3\mu\nu} - \frac{1}{4} g^2 B_{\mu\nu} B^{\mu\nu}$$

Get $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ for A_μ as above.

This is all just a change of basis:

$$\begin{pmatrix} W^1_\mu \\ W^2_\mu \\ W^3_\mu \\ B_\mu \end{pmatrix} \rightarrow \begin{pmatrix} W^{+/-}_\mu \\ \hline Z_\mu \\ A_\mu \end{pmatrix}$$

Gauge eigenstates — Mass eigenstates

$$Q = T^3 + Y$$

Justify:

Study arbitrary $\left[D_\mu = \partial_\mu - i g W^i_\mu \tau^i - i g' Y B_\mu \right]$

Perform change of basis:

$$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}} \left(W^+_\mu \tau^+ + W^-_\mu \tau^- \right) - \frac{i Z_\mu}{\sqrt{g^2 + g'^2}} \left(g^2 T^3 - g'^2 Y \right)$$

$$- \frac{i g g'}{\sqrt{g^2 + g'^2}} A_\mu \left(T^3 + Y \right)$$

$$\tau^+ = \frac{\sigma^1 - i\sigma^2}{2\sqrt{2}}, \quad \tau^- = \frac{\sigma^1 + i\sigma^2}{2\sqrt{2}}$$

Original field had $SU(2) \times U(1)$ charges.

Now reexpress D_μ using mass eigenstates for vectors,

coupling to $W^{+/-}$: $\frac{g}{\ }$.

Now reexpress D_μ using mass eigenstates for vectors,

coupling to $W^{+/-}$: $\frac{g}{\sqrt{2}}$,

Z : $\frac{g^2 T^3 - g'^2 Y}{\sqrt{g^2+g'^2}}$

A : $\frac{g g'}{\sqrt{g^2+g'^2}} \equiv e$, QED gauge coupling

$U(1)_{em}$ Charge $\equiv T^3 + Y$

Check: $u_R \sim (3, 1, 2/3)$

$d_R \sim (3, 1, -1/3)$

$Q_L \sim (3, 2, 1/6)$ give correct up + down-quark EM charges

Define mixing angle

$s_w = \frac{g}{\sqrt{g^2+g'^2}}$, $c_w = \frac{g}{\sqrt{g^2+g'^2}}$, $g = \frac{e}{s_w}$

Key params of SM: weak mixing angle.

Useful to consider:

$\sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}$

Mixing W^3_μ + B_μ to become Z_μ + A_μ.

Two limits:

$g \to 0$	$g' \to 0$
$\Downarrow$	$\Downarrow$
vev of Higgs only breaks $U(1)$	vev is only felt by $SU(2)$ gauge bosons
Z boson aligned with B, ✓	Z boson aligned w/ W^3 ✓
A aligned with W^3. ✓	A aligned B ✓

$Z_\mu = (-g W^3_\mu + g' B_\mu)$

$$Z_\mu = \frac{(-g W^3_\mu + g' B_\mu)}{\sqrt{g^2+g'^2}}$$

$$A_\mu = \frac{(g' W^3_\mu + g B_\mu)}{\sqrt{g^2+g'^2}}$$

θ_W tells you how much Higgs vev is distributed between W^3_μ & B_μ.

$\sin^2\theta_W \simeq 0.23$

Fermion masses & CKM.

Higgs vev also gives mass to fermions.

$$\mathcal{L} = -y_u \bar{Q}_L \tilde{H} u_R - y_d \bar{Q}_L H d_R - y_e \bar{L}_L H e_R + h.c.$$

$$\tilde{H} = i\sigma_2 H^* = \frac{1}{\sqrt{2}}\begin{pmatrix} v+h^0 - iG^0 \\ -G^- \end{pmatrix}$$

$$\mathcal{L} \supset -\frac{y_u}{\sqrt{2}}(v+h_0)\bar{u}_L u_R - \frac{y_d}{\sqrt{2}}(v+h_0)\bar{d}_L d_R - \frac{y_e}{\sqrt{2}}(v+h_0)\bar{e}_L e_R + h.c.$$

Use global symmetries $U(3)_{Q_L}, U(3)_{u_R}, U(3)_{d_R}, U(3)_{L_L}, U(3)_{e_R}$ to diagonalize all y_u, y_d, y_e:

$$\mathcal{L} \supset \bar{e}_L U^\dagger_{e_L} U_{e_L} y_e U^\dagger_{e_R} U_{e_R} e_R \frac{1}{\sqrt{2}}(v+h_0)$$

Choose $U_{e_L} y_e U^\dagger_{e_R} = y_e^{diag}$

then redefine $U_{e_R} e_R = e_R^{mass}$, $U_{e_L} e_L = e_L^{mass}$

So, masses are diagonalized.

$m_u = y_u^{diag} v \quad m = y^{diag} v \quad y^{diag}$.

... are diagonalized.

$$m_e = \frac{y_e^{diag} v}{\sqrt{2}}, \quad m_u = \frac{y_u^{diag} v}{\sqrt{2}}, \quad m_d = \frac{y_d^{diag} v}{\sqrt{2}}$$

Predict Higgs Yukawa interactions are also diagonal.

Study change of basis in covariant kinetic term:

$$\bar{Q}_L i\not{D} Q_L + \bar{u}_R i\not{D} u_R + \bar{d}_R i\not{D} d_R + \text{leptons}$$

Isolate $\bar{u}_L i\not{\partial} u_L \to \bar{u}_L U_{u_L}^\dagger U_{u_L} i\not{\partial} U_{u_L}^\dagger U_{u_L} u_L$

$$= \bar{u}_L^{mass} i\not{\partial} u_L^{mass}$$

+ same for all fields.

$$(\bar{u}_L \; \bar{d}_L)\begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix}\begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$\bar{u}_L \not{Z} g_{eff} u_L$ is same.

But $\bar{Q}_L i\not{D} Q_L \supset \underbrace{(\bar{u}_L ; \bar{d}_L)}_{1\times 2} i \underbrace{\left(\frac{g}{\sqrt{2}} W_\mu^+ \sigma^+ \gamma^\mu + \frac{g}{\sqrt{2}} W_\mu^- \sigma^- \gamma^\mu\right)}_{2\times 2} \underbrace{\begin{pmatrix} u_L \\ d_L \end{pmatrix}}_{2\times 1}$

writing out SU(2) indices

$$= \bar{u}_L \frac{ig}{\sqrt{2}} W_\mu^+ \gamma^\mu d_L + \bar{d}_L \frac{ig}{\sqrt{2}} W_\mu^- \gamma^\mu u_L$$

$$= (\bar{d}_L W_\mu^- \gamma^\mu ; \bar{u}_L W_\mu^+ \gamma^\mu)\begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

Now basis change to mass eigenstates for u_L^m, d_L^m:

$$= \bar{u}_L U_{u_L}^\dagger U_{u_L} \frac{ig}{\sqrt{2}} W_\mu^+ \gamma^\mu U_{d_L}^\dagger U_{d_L} d_L + \bar{d}_L U_{d_L}^\dagger U_{d_L} \frac{ig}{\sqrt{2}} W_\mu^- \gamma^\mu \cdot U_{u_L}^\dagger U_{u_L} u_L$$

$$= \bar{u}_L^m U_{u_L} U_{d_L}^\dagger \frac{ig W_\mu^+}{\sqrt{2}} \gamma^\mu d_L^m + \bar{d}_L^m U_{d_L} U_{u_L}^\dagger \frac{ig}{\sqrt{2}} W_\mu^- \gamma^\mu u_L^m$$

Expressed in terms of mass eigenstates, the $W^{+/-}$ interactions are <u>not</u> diagonal.

the $W^{+/-}$ interactions are <u>not</u> diagonal.

$$U_{u_L} U_{d_L}^\dagger = V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo-Kobayashi-Maskawa

3 mixing angles, 1 $\cancel{CP}$ phase.

Charged currents not diagonal $\exists$ provide all flavor violation is SM.

Analogous matrix for neutrinos

PMNS: Pontecorvo-Maki-Nagawa-Sakata

Summarize:

Z & Higgs interactions are flavor-conserving @ tree-level.

No FCNCs @ tree-level in SM.

$W^{+/-}$ provides all flavor violation in SM.

End of regular lecture series.

Next week: special topic lectures.

Neutrino physics

Flavor physics, Electroweak Hamiltonian

Top physics, Bottom, τ.

* Higgs

* New gauge groups

*** DM

**** Hierarchy problem

* BRST symmetry

* Gauge-gravity duality

☆ Gauge-gravity duality

9:00 am @ Friday office hours.