Lecture 15, June 24, 2020

Wednesday, June 24, 2020 2:17 PM

Last time: finished up ghosts in Y-M theory.

Today: Close off QCD.
- Start EW theory
- Goldstone's theorem
- Spontaneous symmetry breaking + Higgs mechanism.

Office hours: Friday 9 am

Finishing QCD

$$\beta(g) = \frac{-g^3}{16\pi^2}\left(\frac{11}{3}C_2(G) - \frac{4}{3}n_f\, C(r_f)\right)$$

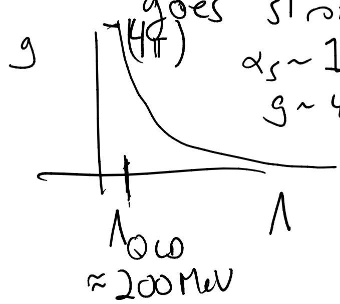
$C_2(G) = N$ for $SU(N)$ theory

Assuming $r_f = \square$ ("fundamental"), $C(r_f) = \frac{1}{2}$

In QCD, $\beta(g) = \frac{-g^3}{16\pi^2}\left(11 - \frac{2}{3}n_f\right)$

For $n_f < \frac{33}{2}$, $\beta < 0$.

Theory is asymptotically free. Coupling runs to smaller values in far UV. Conversely, at small energies, coupling goes strong. Cannot solve strongly coupled theory.



Perturbation theory breaks down.

QCD below Λ_{QCD} becomes a theory of confined objects

Dofs: baryons (antisymmetric contractions of 3 quarks)
+ mesons (quark-antiquark bound states).

Baryons:

Λ (uds)

p (uud)

n (udd)

$\Delta^{I=1/2}$ (uud)

Ω (sss)

Δ^{++} (uuu)

Σ

Ξ

Mesons

$\pi^{+/-}, \pi^0$ (u,d mesons)

$K^{+/-}, K^0, \overline{K^0}$ (strange mesons)

$\underbrace{K^0, \overline{K^0}}_{K_L, K_S}$

$B^{+/-}_{u/c}, B^{0/\bar{0}}_{d/s}$ (bottom mesons)

η, ($s\bar{s}$ scalar + $u\bar{u}/d\bar{d}$)

ρ ($u\bar{u}/d\bar{d}$ vector)

$D^{+/-}, D^0$ (charm mesons)

J/ψ $(c\bar{c})$

Particle physics history: produce each particle + measure properties.

baryon: $\epsilon_{abc}\, q^a q^b q^c$ vs. meson $\bar{q}_a q_a$

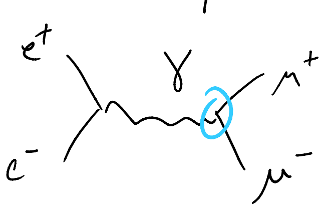
Both are color singlets.

We have not seen colored objects in Nature.

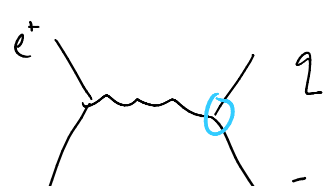
When we produce quarks + gluons at high energies, their color is radiated from final state gluons + pull other colored objects out of vacuum to become color singlet.

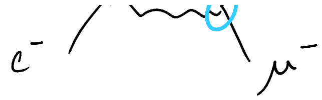
How do we tell that color is $SU(3)_c$ symmetry?

Study $R = \dfrac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ $\Rightarrow$ demonstrates that quarks have multiplicity of 3 that is not accounted for by rescaling.

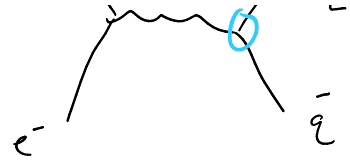


vs.





vs.



$-ieQ_{muon}\gamma^\mu$

$(-ieQ_q\gamma^\mu)$

$$R \equiv \frac{\sigma}{\sigma} \propto \frac{3\Sigma(Q_q)^2}{Q_{muon}^2}$$

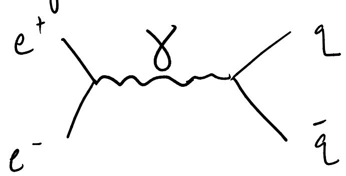
sum over $q\bar{q}$ final states:
sum over spin + multiplicity,
take into account that quarks
have 3 colors.

Way how protons + neutrons break up into constituents of
colored objects ⇒ parton distribution fcns. $\boxed{\text{PDFs}}$

Radiation of gluons (and photons) is governed by
Altarelli-Parisi splitting fcns. ⇒ Parton showers,
soft-collinear effective theory.

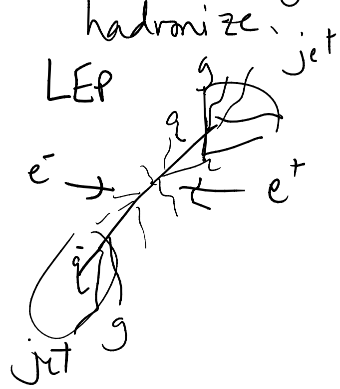
Advertise: Chiral Effective Lagrangian from condensation
of quarks ⇒ studied in Chirality + Gauge Theories.

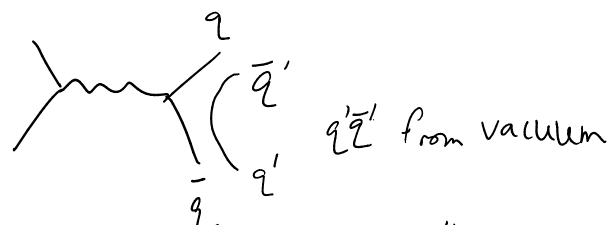
In practice:

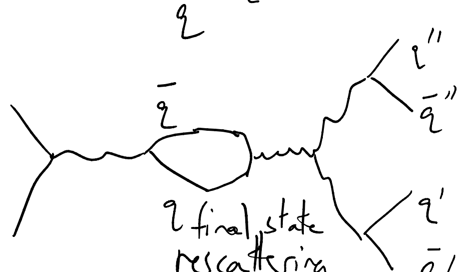


At high energies,
quarks travel
macroscopic distance
before fragment +
hadronize.

LEP

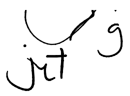


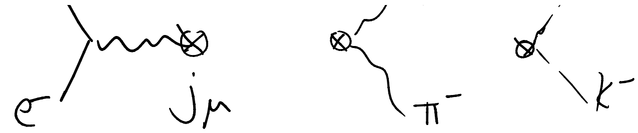




All of these diagrams at low energies
are non-perturbative.

Apply form factors:





Need from data or theory!

$$j_\mu(Q^2) = f_\pi \pi^+(q_1)\pi^-(q_2) + f_K K^+(q_1) K^-(q_2) + \cdots$$

QFT 3:
Standard Model
+ EW Theory.

$K\bar{K}$ mixing
CPV in SM
Flavor physics in SM
Neutrino physics
Top physics, bottom physics
Higgs physics
Oblique corrections: S, T, U
Collider physics

Return 3:08 pm

Standard Model:
Chiral gauge theory.
LH + RH fermions transform differently under electroweak gauge group.

$$\mathcal{L}_{gauge} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$$\mathcal{L}_{Kinetic} = |D_\mu H|^2 + \bar{Q}_L i \not{D} Q_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R + \bar{L}_L i \not{D} L_L + \bar{e}_R i \not{D} e_R + (\bar{N} i \not{D} N)$$

$$\mathcal{L}_{Yukawa} = -y_u \bar{Q}_L \tilde{H} u_R - y_d \bar{Q}_L H d_R - y_e \bar{L}_L H e_R + h.c. + (-y_\nu \bar{L}_L \tilde{H} N + h.c.)$$

$$\mathcal{L}_{scalar} = -V(H) = -\left(\mu^2 |H|^2 + \lambda |H|^4\right)$$

For EWSB, electroweak sym. breaking, $\mu^2 < 0$.

$$\mathcal{L}_{\cancel{CP}} = \frac{+\theta g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + \frac{\theta_2 g^2}{32\pi^2} W^i_{\mu\nu} \tilde{W}^{i\mu\nu} + \frac{\theta_Y g'^2}{16\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu}$$

$$\mathcal{L}_N = -m_N \bar{N} N$$

Quantum numbers: $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge theory

$Q_L \sim (3, 2, 1/6)$ $\quad$ $L_L \sim (1, 2, -1/2)$ $\quad$ $N \sim (1, 1, 0)$

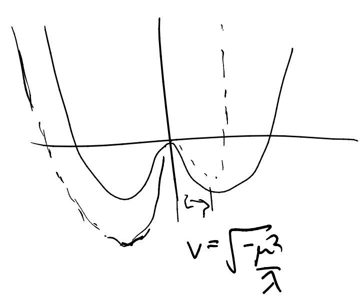
$u_R \sim (3, 1, 2/3)$ $\quad$ $e_R \sim (1, 1, -1)$

$d_R \sim (3, 1, -1/3)$ $\quad$ $H \sim (1, 2, 1/2)$

Major open questions:

θ - QCD, ν - masses (Dirac vs. Majorana).
Hierarchy problem.
Dark matter.
+ many more

First topic: mechanism of spontaneous symmetry breaking.
Higgs potential generates a non zero vacuum expectation value for field.



$$V(H) = \mu^2 |H|^2 + \lambda |H|^4, \quad \mu^2 < 0$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ \\ v + h_0 + i G_0 \end{pmatrix}$$

G^+ is complex scalar Goldstone
G^0 is pseudoreal scalar Goldstone
h_0 is real scalar field.
v is vev, determined by min. cond. of $V(H)$

$\frac{\partial V}{\partial H} = 0$, $\frac{\partial^2 V}{\partial H^2} > 0$ for stable vacuum.

$\Rightarrow \left.\frac{\partial V}{\partial h}\right| = 0$ after vev expansion on ∂H^2 … for stable vacuum.

$$\Rightarrow \frac{\partial V}{\partial h_0}\bigg|_{h_0=0} = 0 \quad \text{after vev expansion}$$

Solve $\Rightarrow V(H) \supset \frac{\mu^2}{2} 2 v h_0 + \frac{\lambda}{4} 4 v^3 h_0 \Rightarrow \quad v\mu^2 + \lambda v^3 = 0$

$$v^2 = -\frac{\mu^2}{\lambda}$$

Key: Lagrangian and equations of motion still obey gauge symmetry. Expansion around vacuum solution to spont. break gauge symmetry.

Goldstone's theorem:

Spontaneous breaking of continuous symmetries leads to massless particles. (P+S 11.1)

Every generator = massless particle.

Pf: $V(\phi)$ has vevs ϕ_0^a.

$$\frac{\partial V}{\partial \phi^a}\bigg|_{\phi^a = \phi_0^a} = 0.$$

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi - \phi_0^a)(\phi - \phi_0^b)\left(\frac{\partial^2}{\partial \phi^a \partial \phi^b} V\right)_{\phi_0} + \ldots$$

Now, $\frac{\partial^2}{\partial \phi^a \partial \phi^b} V = m_{ab}^2$.

Note $\phi^a \to \phi^a + \alpha \Delta^a(\phi)$ symmetry.

$$V(\phi^a) = V(\phi^a + \underbrace{\alpha \Delta^a(\phi)}_{\delta\phi^a})$$

$$\Delta^a(\phi) \frac{\partial V(\phi)}{\partial \phi^a} = 0$$

Get
$$0 = \left(\frac{\partial \Delta^a}{\partial \phi^b}\right)\bigg|_{\phi_0} \underbrace{\left(\frac{\partial V}{\partial \phi^a}\right)\bigg|_{\phi^0}}_{=0} + \Delta^a(\phi_0)\left(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b}\right)\bigg|_{\phi^0}$$

Two possibilities:

① $\Delta^a(\phi) = 0$, then symmetry is respected by ground state … $(\partial^2 V$ …

(1) $\Delta^a(\phi) = 0$, then symmetry is respected by ground state + $\left(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b}\right)\Big|_{\phi_0}$ is unconstrained.

(2) If $\Delta^a(\phi) \neq 0$, then $m^2_{ab} = 0$, so spont. broken symmetry has massless mode.

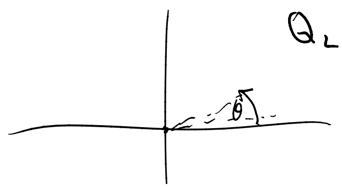
Under $U(1)_Y$, $H \to e^{i g' \cdot Y(H) \theta} H$

$Y(H) = 1/2$

$$H \to e^{i \frac{g'}{2}\theta} H \to \begin{pmatrix} e^{i g' \frac{\theta}{2}} & 0 \\ 0 & e^{i g' \frac{\theta}{2}} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ \\ v + h + i G^0 \end{pmatrix}$$

$$Q_L \to e^{i \frac{g'}{6}\theta} Q_L$$

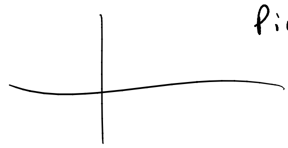
top down



$SU(2)_L$, $H_i \to U_{ij} H_j$

$U_{ij} = \sum a_i \tau_i$, $\sum |a_i|^2 = 1$

Picture is harder.



Distinction on how to apply Goldstone's theorem when continuous symmetry is gauged vs. global

In gauged theory, the gauge bosons "eat" the Goldstone modes + become massive degrees of freedom,

before: massless gauge bosons have 2 dof, transverse pols.

after: massive gauge bosons have 2+1 dof, transverse + longitudinal pols.

Goldstone mode

In global theory, Goldstone mode is explicit massless dof.

——— explicit massless dof.

Quantization in theories w/ spont. symmetry breaking.

Abelian vs. non-Abelian quantization
↳ needed ghosts.

For both kinds of theories, we modify gauge-fixing term to include ξ-dependence: to remove mixing terms between Goldstone dofs + gauge dofs.

$$Z = C' \int \underset{}{DA}\, \underset{\substack{\uparrow \\ \text{Higgs}}}{Dh}\, \underset{\substack{\uparrow \\ \text{Goldstone}}}{D\varphi}\, \exp\left(i \int d^4x \left[\mathcal{L}(A, h, \varphi) - \tfrac{1}{2}(G^2) \right] \det\left(\frac{\delta G}{\delta \alpha}\right)\right)$$

Recall det gave ghost Lagrangian

Now use R_ξ gauge

$$G = \frac{1}{\sqrt{\xi}} \left(\partial_\mu A^\mu - \xi e v \varphi \right) \quad \text{in } \underline{\text{Abelian } \overset{\text{Higgs } U(1)}{\text{Higgs}} \text{ model}}.$$

Then, $\mathcal{L} - \frac{1}{2} G^2$

$$= -\tfrac{1}{2} A_\mu \left(-g^{\mu\nu}\partial^2 + \left(1 - \tfrac{1}{\xi}\right) \partial^\mu \partial^\nu - (ev)^2 g^{\mu\nu} \right) A_\nu$$
$$+ \tfrac{1}{2} (\partial_\mu h)^2 - \tfrac{1}{2} m_h^2 h^2 + \tfrac{1}{2} (\partial_\mu \varphi)^2 - \tfrac{\xi}{2} (ev)^2 \varphi^2$$

Vector propagator:

$$\underset{\mu \quad \overleftarrow{k} \quad \nu}{A} \qquad \frac{-i}{k^2 - m_A^2} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2 - \xi m_A^2} (1 - \xi) \right)$$

Goldstone mass is gauge dependent: $m_\varphi^2 = \xi (ev)^2$

Vector boson mass $m_A^2 = (ev)^2$

We get $\displaystyle\sum_{3 \text{ pols.}} \epsilon^\mu \epsilon^{\nu *} = -g^{\mu\nu} + \frac{q^\mu q^\nu}{m_A^2}$

Choice of ξ is arbitrary.

Popular choices:

$\xi = 0$: Lorentz gauge

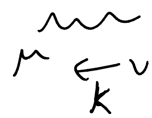
$\xi = 1$: Feynman gauge

$$\frac{-i\,g^{\mu\nu}}{k^2 - m_A^2}$$

$$\frac{i}{k^2 - m_A^2}$$

explicit dof matches

$\xi = \infty$: Unitarity gauge.



$$\frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_A^2}\right)$$