Lecture 15 Notes.

QFT II.

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Last time: Finished $\Pi^{\mu\nu}(q^2)$ in QCD + discussed β-fcn, asymptotic freedom.

Today: Finish QCD + begin Standard Model + Electroweak Symmetry; Higgs Mechanism + Goldstone Theorem.

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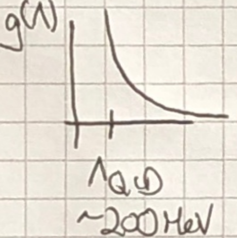
Recall QCD β-fcn:

$$\beta(g) = \frac{-g^3}{16\pi^2}\left(\frac{11}{3} C_2(G) - \frac{4}{3} n_f C(r_f)\right)$$

$C_2(G) = N_c$, $\quad C(r_f) = \frac{1}{2}$ for r_f = fundamental

$$\Rightarrow \beta(g) = \frac{-g^3}{16\pi^2}\left(11 - \frac{2n_f}{3}\right)$$



So, for $n_f < \frac{33}{2} = 16.5$, we retain asymptotic freedom + theory is weakly coupled in UV + strongly coupled in IR.

Thus, free, colored quarks have not been observed.

Deep inelastic scattering (DIS) shows nature of proton, neutron to be composite particles, not fundament (P+S, 17.3). We have two ways to form color singlets from quarks: baryons (antisymmetric contraction of 3 colors) + mesons (anti-fundamental + fundamental contraction)

Examples:

$\epsilon^{abc} q_a q_b q_c$ Baryons: $p = (uud)$, $n = (udd)$, $\Lambda, \Delta, \Omega, \Delta^{++}, \Sigma, \Xi$

$\bar{q}_a q_a$ Mesons: $\pi^{+/-}$, π^0, $K_L, K_S, K^{+/-}$, $B^{+/-}$, B_d^0, B_s^0,

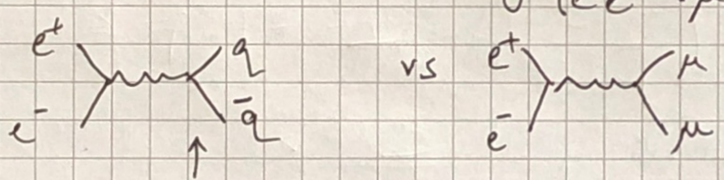
$D^0, D^{+/-}, \eta, J/\psi$

When colored particles are produced at high energies, they radiate gluons (+ photons) before losing enough energy + confining into mesons.
Governed by Altarelli - Parisis splitting (P+S 17.5).
To form color singlets, they pull colored objects from vacuum.

The opposite direction: accelerate composite objects to high energies + their constituents interact. The parton content of composite objects is known as the Parton Distribution Function (PDF).

Finally, we can tell there are 3 colors (+ not 4, for example) by studying

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



extra t^a + charge.

Electroweak theory + full Standard Model Lagrangian
↳ Glashow, Weinberg, Salam

The Electroweak gauge symmetry is distinct from QCD because the fermions are <u>chiral</u>, i.e. LH + RH fermions tranform differently.

Gauge group is $SU(3)_c \times SU(2)_L \times U(1)_Y$
↳ Hypercharge

Fields:

$Q_L \sim (3, 2, 1/6)$ $\quad L_L \sim (1, 2, -1/2)$

$u_R \sim (3, 1, 2/3)$ $\quad e_R \sim (1, 1, -1)$

$d_R \sim (3, 1, -1/3)$ $\quad [N_R \sim (1, 1, 0)] \leftarrow$ RH neutrino

$H \sim (1, 2, 1/2)$

$$\mathcal{L}_{gauge} = -\frac{1}{4} B_{\mu\nu}B^{\mu\nu} - \frac{1}{4} W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

$i = 1, \ldots 3$
$a = 1, \ldots 8$

$$\mathcal{L}_{Kinetic} = |D_\mu H|^2 + \bar{Q}_L i \not{D} Q_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R + \bar{L}_L i \not{D} L_L + \bar{e}_R i \not{D} e_R + [\bar{N} (i \not{\partial} N)]$$

Each matter field: $Q_L, u_R, d_R, L_L, e_R, N$ has 3 generations.

$$\mathcal{L}_{Yukawa} = -y_u \bar{Q}_L \tilde{H} u_R - y_d \bar{Q}_L H d_R - y_e \bar{L}_L H e_R + h.c.$$
$$(-y_\nu \bar{L}_L \tilde{H} N + h.c.)$$

$$\mathcal{L}_{scalar} = -V(H) = -(\mu^2 |H|^2 + \lambda |H|^4)$$

with $\mu^2 < 0$ for EWSB.

$$\mathcal{L}_{\cancel{CP}} = \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} + \theta_2 \frac{g^2}{32\pi^2} W^i_{\mu\nu} \tilde{W}^{i\mu\nu} + \theta_1 \frac{g'^2}{16\pi^2} B_{\mu\nu} \tilde{B}^{\mu\nu}$$

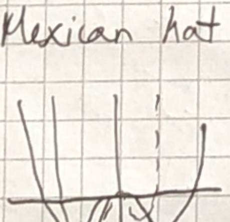
g_s: coupling const. for $SU(3)_c$
g: " " " $SU(2)_L$
g': " " " $U(1)_Y$

$$\mathcal{L}_N = -m_N \bar{N}^c N$$

Majorana mass term for neutrinos.

Many open questions to possible terms in $\mathcal{L}$:
θ-QCD, ν masses, Majorana vs. Dirac, hierarchy problem, dark matter.

First topic:
Mechanism of spontaneous symmetry breaking
Higgs potential generates non-zero vacuum expectation value (vev) for field.
Mexican hat potential



$$V(H) = \mu^2 |H|^2 + \lambda |H|^4, \quad \mu^2 < 0$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} G^+ \\ v + h_0 + i G_0 \end{pmatrix}$$

Use global $SU(2)$ rotation to put v in lower component

G^+ is complex scalar Goldstone

G^0 is pseudoreal scalar Goldstone

h_0 is real Higgs field

v is vev, determined by minimization of V:

$$\frac{\partial V}{\partial H} = 0 \quad + \quad \frac{\partial^2 V}{\partial H^2} \geq 0 \text{ for stable vacuum.}$$

$$\left.\frac{\partial V}{\partial h_0}\right|_{h_0=0} = 0$$ after inserting vev (linear term in Taylor expansion vanishes)

$$V \supset \frac{\mu^2}{2} 2vh_0 + \frac{\lambda}{4} 4v^3 h_0$$

$$\Rightarrow v\mu^2 + \lambda v^3 = 0 \Rightarrow v^2 = -\frac{\mu^2}{\lambda}$$

Key point: Lagrangian + EOMs still obey gauge symmetry. Spontaneous breaking comes only because we expand vacuum solution.

Need to identify Goldstones + their role in SSB.
Goldstone theory + Goldstone theorem.

Spontaneous breaking of continuous symmetries leads to massless particles. (P+S 11.1)

Every generator = massless particle.

Proof: $V(\phi)$ has vevs ϕ_0^a.

$$\left.\frac{\partial V}{\partial \phi^a}\right|_{\phi^a=\phi_0^a} = 0$$

Taylor expand

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi-\phi_0^a)(\phi-\phi_0^b)\underbrace{\left(\frac{\partial^2}{\partial\phi^a\partial\phi^b} V\right)_{\phi_0}}_{= m^2_{ab}}$$

Note $\phi^a \to \phi^a + \alpha\Delta^a(\phi)$ is symmetry.

For ϕ^a const., kinetic deriv. vanishes.

Then, $V(\phi^a) = V(\phi^a + \alpha\, \Delta^a(\phi))$ by symmetry.

$$\Rightarrow \Delta^a(\phi) \frac{\partial V(\phi)}{\partial \phi^a} = 0$$

Get $$0 = \left(\frac{\partial \Delta^a}{\partial \phi^b}\right)\Bigg|_{\phi^0} \underbrace{\left(\frac{\partial V}{\partial \phi^a}\right)\Bigg|_{\phi^0}}_{=0} + \Delta^a(\phi_0)\left(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b}\right)\Bigg|_{\phi^0}$$

If $\Delta^a(\phi_0) = 0$, then symmetry is respected by ground state and $\left(\frac{\partial^2 V}{\partial \phi^a \partial \phi^b}\right)\Big|_{\phi^0}$ is unconstrained.

If $\Delta(\phi_0) \neq 0$, then $m^2_{ab} = 0$, so spont. broken symmetry has massless modes. ✓

In gauge theory, both Abelian + non-Abelian, gauge bosons "eat" Goldstone modes of Higgs field to become massive. Dof: 2 transverse, 1 longitudinal.

Quantization requires usual gauge-fixing in functional integral. We will demonstrate that the gauge fixing procedure establishes the direct connection between Goldstone + gauge boson dofs.

Abelian path integral

$$Z = \int \mathcal{D}A \underbrace{\mathcal{D}h}_{\text{Higgs}} \underbrace{\mathcal{D}\varphi}_{\text{Goldstones}}\, e^{iS[A,h,\varphi]}$$

Abelian Higgs model

$$\phi = \frac{1}{\sqrt{2}}(\phi^1 + i\phi^2) = \frac{1}{\sqrt{2}}(v + h + \varphi)$$

$$Z = C\int \mathcal{D}A\, \mathcal{D}h\, \mathcal{D}\varphi\, e^{iS[A,h,\varphi]} \cdot \delta(G(A,h,\varphi)) \det\left(\frac{\delta G}{\delta \alpha}\right)$$

Earlier: $G(x) = \partial_\mu A^\mu(x) - \omega(x)$ was the gauge fixing term.

This gave $Z = C' \int \mathcal{D}A\, \mathcal{D}h\, \mathcal{D}\varphi \exp\left[i\int d^4x \left(\mathcal{L}[A,h,\varphi] - \frac{1}{2}G^2\right)\right] \det\left(\frac{\delta G}{\delta \alpha}\right)$

+ det term gave ghost Lagrangian.

Introduce R_ξ gauge:

$$G = \frac{1}{\sqrt{\xi}}\left(\partial_\mu A^\mu - \xi e v \varphi\right)$$

Removes cross terms between A_μ + φ (check!)

$$\mathcal{L} - \frac{1}{2} G^2$$
$$= \frac{-1}{2} A_\mu\left(-g^{\mu\nu}\partial^2 + \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu - (ev)^2 g^{\mu\nu}\right)A_\nu$$
$$+ \frac{1}{2}(\partial_\mu h)^2 - \frac{1}{2}m_h^2 h^2 + \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{\xi}{2}(ev)^2\varphi^2$$

Goldstone mass is gauge-dependent $m_\varphi^2 = \xi (ev)^2$

Vector boson $m_A^2 = (ev)^2$

Vector propagator

A

$\mu \leftarrow \nu$ $\qquad \dfrac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \dfrac{k^\mu k^\nu}{k^2 - \xi m_A^2}(1-\xi)\right)$

$$\mathcal{L}_{ghost} = \bar{c}\left[-\partial^2 - \xi m_A^2\left(1 + \frac{h}{v}\right)\right]c$$

Ghost mass: ξm_A^2, couples to Higgs.

Modification of $\sum_{pols} \epsilon^\mu \epsilon^{\nu *} = -g^{\mu\nu} + \dfrac{q^\mu q^\nu}{m_A^2}$

since we now have 3 polarizations.

The choice of ξ is known as the choice R_ξ:

Gauge choices:

Lorentz: $\xi = 0$

$\mu \leftarrow \nu$ $\qquad \dfrac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \dfrac{k^\mu k^\nu}{k^2}\right)$

Feynman-'t Hooft: $\xi = 1$

$\mu \overset{}{\underset{k}{\leftarrow}} \nu$ $\qquad \dfrac{-i}{k^2 - m_A^2}(g^{\mu\nu})$

$- - \leftarrow - -$ $\qquad \dfrac{i}{k^2 - m_A^2}$

Unitarity: $\xi \to \infty$

$\mu \underset{k}{\leftarrow} \nu$ $\qquad \dfrac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \dfrac{k^\mu k^\nu}{m_A^2}\right)$

Non-Abelian R_ξ:

Same analysis carries through, except more Goldstones.

$$\delta\phi_i = -\alpha^a(x)\,\underbrace{T^a_{ij}\,\phi_j}_{\Delta^a \text{ from before}}$$

ϕ_i is multiplet of real-valued fields.

$$D_\mu\phi_i = \partial_\mu\phi_i + g A^a_\mu T^a_{ij}\phi_j$$

$$\phi_i = v_i + \chi_i(x)$$

Goldstones: field fluctuations along directions spanned by $T^a_{ij} v_j$.

Orthogonal: physical scalars.

Convenient to use $F^a_i = T^a_{ij} v_j$

Not necessarily square.

$$Z = \int DA\,D\chi\; e^{i\int\mathcal{L}[A,\chi]}$$

$$= C'\int DA\,D\chi\, \exp\left[i\int d^4x\,\mathcal{L}[A,\chi] - \frac{1}{2}(G)^2\right] \det\left(\frac{\delta G}{\delta\alpha}\right)$$

Gauge-fixing function:

$$G^a = \frac{1}{\sqrt{\xi}}\left(\partial_\mu A^{a\mu} - \xi g F^a_i \chi_i\right) \qquad \left[\text{cf. } \frac{1}{\sqrt{\xi}}\left(\partial_\mu A^\mu - \xi e v \varphi\right)\right]$$

Then quadratic part of $-\frac{1}{2}G^2$ is

$$= \frac{1}{2} A^a_\mu\left(\frac{1}{\xi}\partial^\mu\partial^\nu\right)A^a_\nu + g\,\partial_\mu A^{a\mu} F^a_i \chi_i - \frac{1}{2}\xi g^2 \left[F^a_i \chi_i\right]^2$$

$$= -\frac{1}{2} A^a_\mu\left(\left[-g^{\mu\nu}\partial^2 + \left(1-\frac{1}{\xi}\right)\partial^\mu\partial^\nu\right]\delta^{ab} - g^2 F^a_i F^b_i g^{\mu\nu}\right)A^b_\nu$$
$$+ \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}\xi g^2 F^a_i F^a_j \chi_i \chi_j$$

Ghost $$\left(\frac{\partial G^a}{\partial\alpha^b}\right) = \frac{1}{\sqrt{\xi}}\left(\frac{1}{g}\left(\partial_\mu D^\mu\right)^{ab} + \xi g\,(T^a v)\cdot T^b(v+\chi)\right)$$

$$\mathcal{L}_{ghost} = \bar{c}^a\left[-\left(\partial_\mu D^\mu\right)^{ab} - \xi g^2\, T^a\phi_0 \cdot T^b(\phi_0+\chi)\right]c^b$$

In electroweak, $\mu \sim\!\sim\!\sim \nu$ ($\xleftarrow{k}$) $= \dfrac{-i}{k^2-m^2}\left(g^{\mu\nu} - \dfrac{k^\mu k^\nu}{k^2-\xi m^2}(1-\xi)\right)$

for $m^2 = m_W^2, m_Z^2, m_A^2 = 0$.

Goldstones: $- - - -$ $\dfrac{i}{k^2-\xi m^2}$ 3 Goldstones

Ghosts $\cdots\cdots$ ($\leftarrow$) $\dfrac{i}{k^2-\xi m^2}$ 4 ghosts match $m_W^2, m_Z^2, m_A^2=0$.