08.128.809 Theoretische Elementarteilchenphysik
Quantum Field Theory II

Homework set 4

Due July 5, 2018
Please note how long it took you to solve each problem!

Question 1 Z boson decay widths. The Z boson is the heavy cousin of the electromagnetic photon that arises from spontaneous breaking of the $SU(2) \times U(1)$ electroweak symmetry by the Higgs field. In this problem, we will calculate the partial widths of the Z boson to Standard Model fermions. In parts B-F, you can assume the fermion masses vanish.

A. From the expressions for the Z current (reproduced from Eqs. 20.79 and 20.80 in Peskin and Schroeder), calculate the Feynman rules for Z couplings to neutrinos, charged leptons, up-type quarks, and down-type quarks.

$$\mathcal{L} = g Z^0_\mu J^\mu_Z \tag{1}$$

$$\begin{aligned} J^\mu_Z = \frac{1}{\cos\theta_W} \Big[& \bar{\nu}_L \gamma^\mu \frac{1}{2} \nu_L + \bar{e}_L \gamma^\mu (-\frac{1}{2} + \sin^2\theta_W) e_L + \bar{e}_R \gamma^\mu (\sin^2\theta_W) e_R \\ & + \bar{u}_L \gamma^\mu (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W) u_L + \bar{u}_R \gamma^\mu (-\frac{2}{3}\sin^2\theta_W) u_R \\ & + \bar{d}_L \gamma^\mu (-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W) d_L + \bar{d}_R \gamma^\mu (\frac{1}{3}\sin^2\theta_W) d_R \Big] \end{aligned} \tag{2}$$

To combine the different chiralities, recall that $f_L = P_L f$ and $f_R = P_R f$ using four-component notation.

B. Calculate the partial width for $Z \to \nu\bar{\nu}$ for one flavor of neutrinos.

C. Calculate the partial width for $Z \to \ell\bar{\ell}$ for one flavor of charged leptons.

D. Calculate the partial width for $Z \to u\bar{u}$ for one flavor of up-type quarks.

E. Calculate the partial width for $Z \to d\bar{d}$ for one flavor of down-type quarks.

F. Since $m_Z = 91.2$ GeV and $m_t = 173.4$ GeV, the Z boson cannot decay into two top quarks. All the other possible decays to SM fermions are allowed, though. Given that we have three generations of each type of fermion (and removing one multiplicity for up-type quarks because of kinematics), what is the total Z decay width? And what is the Z proper lifetime?

Question 2 Perturbative unitarity of electroweak scattering in the Standard Model and demonstration of Goldstone boson equivalence. (This unitarity calculation is the basis for the construction of the LHC in CERN and the prediction of the Higgs boson. The original paper is by B. Lee, C. Quigg, and H. Thacker, "Weak Interactions at Very High-Energies: The Role of the Higgs Boson Mass," PRD 16 (1977) 1519.) We start with the abbreviated Lagrangian of electroweak gauge interactions,

$$\begin{aligned}\mathcal{L} \supset\ & \frac{1}{2}\left(\partial_\mu h\right)^2 - \frac{1}{2}m_h^2 h^2 + 2\frac{m_W^2}{v} h W_\mu^+ W^{\mu,-} + \frac{m_Z^2}{v} h Z_\mu Z^\mu \\ & - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}m_Z^2 Z^\mu Z_\mu \\ & + ie\cot\theta_w (Z_{\mu\nu}W_\mu^+ W_\nu^- - W_{\mu\nu}^+ Z_\mu W_\nu^- + W_{\mu\nu}^- Z_\mu W_\nu^+) \\ & + ie(F_{\mu\nu}W_\mu^+ W_\nu^- - W_{\mu\nu}^+ A_\mu W_\nu^- + W_{\mu\nu}^- A_\mu W_\nu^+) \\ & + \frac{1}{2}\frac{e^2}{\sin^2\theta_w}(W_\mu^+ W_\mu^+ W_\nu^- W_\nu^- - W_\mu^+ W_\mu^- W_\nu^+ W_\nu^-), \end{aligned} \quad (3)$$

where we have adopted unitary gauge.

A. Draw all Feynman diagrams for tree-level scattering of $W^+W^- \to W^+W^-$ and $W^+W^+ \to W^+W^+$.

B. In unitary gauge, the gauge boson propagator for a massive vector is $\frac{-i}{q^2 - m^2}(g^{\mu\nu} - \frac{q^\mu q^\nu}{m^2})$. Write out the matrix elements for $W^+W^+ \to W^+W^+$.

C. Adopting R_ξ gauge with $\xi = 1$ (Feynman gauge), what is the expression for the massive gauge propagator? And what are the new diagrams for $W^+W^+ \to W^+W^+$ scattering? Comparing between (B) and (C), you can see diagramatically the equivalence between the *longitudinal* modes of the vector boson propagator and the *Goldstone* modes that appear in Feynman gauge.

Bonus, D. If you sum the matrix elements for $W^+W^+ \to W^+W^+$ but ignore the Higgs contribution, show that the leading E^2 contribution in the matrix element cancels because of gauge coupling equivalence between 3-pt. and 4-pt. vertices. Note there is still a growth in the matrix element at large energies that scales linearly with energy.

Bonus, E. Show that the linear energy growth in part (D) is cancelled by the Higgs-mediated diagram. (In Lee, Quigg, and Thacker, by keeping the Higgs mass a free parameter and imposing the requirement of partial wave unitarity, the Higgs mass can be bounded to be below about 800 GeV.)