08.128.809 Theoretische Elementarteilchenphysik
Quantum Field Theory II

Homework set 4

Due June 21, 2018
Please note how long it took you to solve each problem!

Question 1 Electron electric dipole moment. *Remark: In the Standard Model, the EDM of the electron is expected to be tiny (smaller than* 10^{-38}*e cm), and the leading contribution in the limit of massless neutrinos arises from 4-loop diagrams. The current experimental sensitivity is at the level of* 10^{-29}*e cm, given by the ACME collaboration.* Given the abbreviated SM Lagrangian

$$\begin{aligned}\mathcal{L} = &\bar{e}(i\not{D} - m_e)e + (\frac{y_e}{\sqrt{2}}\bar{e}_L h e_R + \text{h.c.}) \\ &+ \bar{t}(i\not{D} - m_t)t + (\frac{y_t}{\sqrt{2}}\bar{t}_L h t_R + \text{h.c.}) \\ &- \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}m_Z^2 Z^\mu Z_\mu + \frac{1}{2}m_h^2 h^2\end{aligned} \tag{1}$$

where

$$\begin{aligned}D_\mu e &\equiv \left(\partial_\mu + ieA_\mu - i\frac{g}{4\cos\theta_w}Z_\mu\left((-1+4\sin^2\theta_w)+\gamma_5\right)\right)e \\ D_\mu t &\equiv \left(\partial_\mu - i\frac{2}{3}eA_\mu - i\frac{g}{4\cos\theta_w}Z_\mu\left(\left(1-\frac{8}{3}\sin^2\theta_w\right)-\gamma_5\right)\right)t\end{aligned} \tag{2}$$

and

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \; , Z_{\mu\nu} \equiv \partial_\mu Z_\nu - \partial_\nu Z_\mu \; , \tag{3}$$

draw all 1PI diagrams for electron coupling to a photon at tree, one-loop, and two-loop level. Nomenclature: in this Lagrangian, the Higgs field is h, e is the electron, t is the top quark, A_μ is the photon, and Z_μ is the Z-boson. *Remark: the two-loop diagrams with internal* h *and* Z *propagators are called "Barr-Zee" diagrams, which provide the leading constraint on possible CP-violating phase of the top quark Yukawa coupling.*

Question 2 In beyond the Standard Model physics, an extension of the Standard Model QCD gauge symmetry is natural to consider. In particular, the SM $SU(3)_c$ color symmetry is enhanced to a product group of $SU(3)_1 \times SU(3)_2$ at a high scale, and the diagonal subgroup [where the separate generators of each $SU(3)$ are identified with each other] is the familiar $SU(3)_c$ symmetry of the Standard Model. The massless gluon, which mediates the unbroken $SU(3)_c$ symmetry, then acquires a partner X, which is a massive color octet vector (commonly referred to as an axigluon or coloron). A relevant set of Lagrangian terms is then

$$\mathcal{L} = \bar{t}(i\not{\partial} - m_t)t + g_s(\lambda\bar{t}_L\gamma_\mu t^a X^{a,\mu}t_L + \kappa\bar{t}_R\gamma_\mu t^a X^{a,\mu}t_R) - \frac{1}{4}X^a_{\mu\nu}X^{a,\mu\nu} + m_X^2 X_\mu X^\mu \; . \tag{4}$$

A. Rewrite the Lagrangian coupling for X_μ to t by using $t_L = P_L t$ and $t_R = P_R t$. What is the Feynman rule for X_μ interacting with t?

B. Calculate the decay width for $X \to t\bar{t}$. Here, t transforms in the fundamental representation of the $SU(3)_c$ symmetry and X transforms in the adjoint representation. When evaluating the polarization sum of the matrix element squared, you will need to use

$$\sum_{\text{polarizations}} \epsilon_\mu \epsilon_\nu^* = (-g_{\mu\nu} + \frac{k_\mu k_\nu}{m_X^2}) \ , \tag{5}$$

where k is the momentum of the X particle. (This is the generic form of the modification of the polarization sum when the external vector field is massive. We will discuss this further in the context of the Higgs mechanism and Goldstone boson equivalence, but for background reading, see sections 21.1 and 21.2 of Peskin and Schroeder.)

C. What is the partial width for $X \to t\bar{t}$ if $m_t \to 0$?

D. Assuming the only decay channel is $X \to t\bar{t}$, and for $m_X = 1$ TeV, $m_t = 173$ GeV, $g_s\lambda = g_s\kappa = 0.1$, what is the lifetime of X?

Question 3 Flavor physics is the study of global symmetries and their breaking patterns via interactions. Most commonly, fields are redefined to have *canonical kinetic terms* and *diagonal mass terms*, leaving the interactions, which involve three fields or more, in the most general form. Consider the Lagrangian

$$\mathcal{L} = \bar{\psi}_i (i\not{D}) \psi_i - m_{ij} \bar{\psi}_i \psi_j \ , \tag{6}$$

where the i and j indices run from 1 to N_F. Assume $\not{D}$ is the same for all of the ψ fields.

A. What is the largest symmetry transformation of ψ that leaves $\bar{\psi}_i(i\not{D})\psi_i$ invariant?

B. What is the restriction on the matrix m_{ij} such that the Lagrangian is Hermitian? Tip: it is easiest to add the Hermitian conjugate and then manipulate the terms to have the same field structure. (Recall that Lagrangians must be Hermitian in order to satisfy CPT-invariance.) Given the global symmetry transformation in part A, can m_{ij} be simplified to always have diagonal form?

C. Suppose we added a term, $y_{ij}\phi\bar{\psi}_i\psi_j$, where ϕ is a real scalar field, to the Lagrangian. Assuming the y_{ij} matrix is independent of the m_{ij} matrix, what are the restrictions on y_{ij} such that the Lagrangian is Hermitian? And what does the symmetry transformation performed in part B do to the y_{ij} matrix?