08.128.809 Theoretische Elementarteilchenphysik
Quantum Field Theory II

Homework set 2

Due May 17, 2018
Please note how long it took you to solve each problem!

Question 1 Using the functional method, derive the Feynman rules in momentum space for the following interaction vertices:

A. $\mathcal{L} = g\bar{\psi}\gamma_\mu A^\mu \psi$

B. $\mathcal{L} = y\phi\bar{\psi}\psi +$ h.c.

C. $\mathcal{L} = \lambda\phi^4$

D. $\mathcal{L} = gf_{abc}\partial_\mu A^a_\nu A^{\mu,\ b} A^{\nu,\ c}$. For convenience, define all momenta to flow into the vertex, and assume f_{abc} is totally antisymmetric under interchange of any two [group space] indices.

E, Extra credit. $\mathcal{L} = \frac{1}{4}g^2(f^{eab}A^a_\mu A^b_\nu)(f^{ecd}A^{\mu,\ c}A^{\nu,\ d})$. Again, assume f_{abc} is totally antisymmetric under interchange of any two [group space] indices.

Question 2 Using Feynman rules derived from the Lagrangian:

$$\begin{aligned}\mathcal{L} = &\, i\bar{f}_1\not{\partial} f_1 + i\bar{f}_2\not{\partial} f_2 + i\bar{f}_3\not{\partial} f_3 - m_1\bar{f}_1 f_1 - m_2\bar{f}_2 f_2 - m_3\bar{f}_3 f_3 \\ &- \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) \\ &+ Q_2 e\bar{f}_2\not{A} f_2 + Q_3 e\bar{f}_3\not{A} f_3 + g(\bar{f}_3\not{W}^+ P_L f_1 + \text{ h.c.})\ ,\end{aligned}$$

write the matrix elements corresponding to the diagrams in Fig. 1. Note: You do not need to evaluate any of the matrix elements. Also, because there is no mass term for either vector, you can use the massless vector propagator from QED.

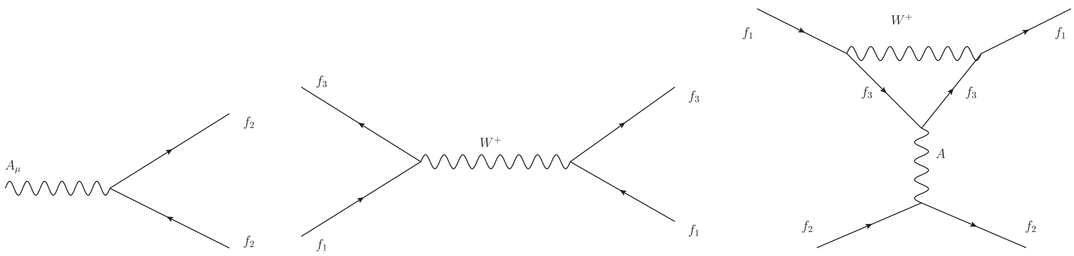

Figure 1: (A.) Vector coupling to two fermions, $A^\mu \to f_2\bar{f}_2$. (B.) Charged current scattering, $f_1\bar{f}_3 \to f_1\bar{f}_3$ via exchange of W^+. (C.) The penguin diagram. You can think of the penguin diagram as a one-loop vertex correction to a vector current where the external vector is allowed to propagate off-shell.

Question 3 Using Feynman rules derived from the Lagrangian:

$$\mathcal{L} = i\bar{f}_1 \partial\!\!\!/ f_1 + i\bar{f}_2 \partial\!\!\!/ f_2 + i\bar{f}_3 \partial\!\!\!/ f_3 - m_1\bar{f}_1 f_1 - m_2\bar{f}_2 f_2 - m_3\bar{f}_3 f_3$$
$$- \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \frac{1}{4}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-)$$
$$+ Q_2 e\bar{f}_2 A\!\!\!/ f_2 + Q_3 e\bar{f}_3 A\!\!\!/ f_3 + g\cos\theta(\bar{f}_3 W\!\!\!\!/^+ P_L f_1 + \text{ h.c.}) + g\sin\theta(\bar{f}_2 W\!\!\!\!/^+ P_L f_1 + \text{ h.c.}) \ ,$$

draw all the diagrams for $f_2\bar{f}_3 \to f_2\bar{f}_3$ scattering at tree-level and one-loop. Be sure to include diagrams that cross internal propagator legs. You should have one diagram, known as the box diagram, as shown here:

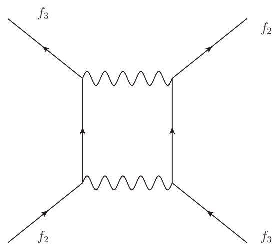

Figure 2: The box diagram.

(This diagram is central to understanding the phenomenology of SM mesons.) For the box diagram in Fig. 2, write the matrix element. Does the loop integral superficially converge or diverge? For all of your diagrams, group them according to the dependence on the gauge couplings e and g. Do all diagrams within each class (diagrams that share the same parametric dependence on e and g) share the same superficial degree of divergence? What physical condition ensures sensitivity to UV divergences vanishes? (Extra credit) If you are ambitious, calculate the leading loop-momentum dependence of each diagram within each class and verify that the UV divergence cancels, leaving only a finite correction (you may need to introduce a mass for each gauge boson to regulate IR divergences).