08.128.809 Theoretische Elementarteilchenphysik
Quantum Field Theory II

Homework set 1

Due April 26, 2018
Please note how long it took you to solve each problem

Question 1 For quantum field theories in 1+1, 2+1, and 5+1 dimensions (# spatial + # time dimensions), give the scaling dimension of scalar fields, fermions, and vector fields. Also, characterize the following Lagrangian terms as super-renormalizable, renormalizable, or non-renormalizable in each $D = 2$, 3, and 6: Yukawa interaction $y\phi\bar{\psi}\psi$, scalar QED $e\partial_\mu\phi^\dagger A^\mu\phi$, "normal" QED $e\bar{\psi}\not{A}\psi$, cubic scalar interaction ϕ^3. (Recall that Lagrangian kinetic terms always involve two spacetime derivatives for bosons and one spacetime derivative for fermions.)

Question 2 Demonstrate Furry's theorem (see problem 10.1 in Peskin and Schroeder) that the one-loop matrix element for three external photons in QED vanishes.

Question 3 Problem 10.2 of Peskin and Schroeder. Starting with the pseudoscalar Yukawa Lagrangian,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\not{\partial} - M)\psi - ig\phi\bar{\psi}\gamma^5\psi \tag{1}$$

with ϕ a real scalar field and ψ is a Dirac fermion. (A) Determine the superficially divergent amplitudes and the Feynman rules for renormalized perturbation theory. Specify the necessary counterterms. You should find a superficially divergent 4ϕ amplitude, which requires a Lagrangian term,

$$\delta\mathcal{L} = \frac{\lambda}{4}\phi^4$$

and a matching counterterm. Are other interactions required? (B) Compute the pole as $d \to 4$ of each counterterm to one-loop order and specify the renormalization conditions.