08.128.165 Theorie 6a, Relativistische Quantenfeldtheorie Quantum Field Theory I

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Homework set 9

Due January 18, 2022 by start of lecture.
Please note how long it took you to solve each problem.

9-1, 25 pts. *Practice with Dirac algebra (part 4).* Verify the following identities. *Note: All of the following are written with u and v spinors, but they are generic for $u \leftrightarrow v$ interchange.*

A, 10 pts. $(\bar{v}\gamma^\mu u)^\dagger = (\bar{u}\gamma^\mu v)$

B, 10 pts. $(\bar{u}\gamma^\mu\gamma^5 v)^\dagger = (\bar{v}\gamma^\mu\gamma^5 u)$

C, 5 pts. Given the answers to parts A and B, what is the Hermitian conjugate of $(\bar{v}\gamma^\mu P_L u)$ and $(\bar{v}\gamma^\mu P_R u)$?

9-2, 10 pts. Show that a given fermion field transforms the same as the covariant derivative of the field under a $U(1)$ symmetry transformation.

9-3, 65 pts. *Bhabha scattering, Peskin and Schroeder problem 5.2.*

A, 10 pts. Given the Feynman rules for QED, draw the two tree-level diagrams for $e^+e^- \to e^+e^-$ scattering, labeling all lines and vertices as necessary.

B, 10 pts. Write the two matrix elements for these diagrams. What is the relative sign between the two diagrams?

C, 20 pts. Setting the electron mass to 0, solve for the differential cross section $d\sigma/d\cos\theta$ for Bhabha scattering. *Hint: Recall that we have identities for Lorentz-contracted γ matrices surrounding other gamma matrices, from HW problem 4-4. Reexpress the kinematic invariants the Mandelstam variables, and note that for massless electrons, $s + t + u = 0$.* You should get the result

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left[u^2\left(\frac{1}{s} + \frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2\right] \tag{1}$$

D, 10 pts. Rewrite the differential cross section in terms of s and the scattering angle $\cos\theta$. What is the leading behavior as $\theta \to 0$, and what feature of the diagrams causes this behavior?

E, 15 pts. For the total cross section, we should impose a cut on $\theta > \theta_0$, with $\theta_0 \ll 1$, since it is impossible to distinguish forward scattering from no scattering at all (in contrast to $e^+e^- \to \mu^+\mu^-$). Integrate the differential cross section with such a cutoff, keeping only terms that are singular or constant in the limit $\theta_0 \to 0$.