

08.128.165 Theorie 6a, Relativistische Quantenfeldtheorie

Quantum Field Theory I

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Homework set 6

Due December 7, 2022 by start of lecture.

Please note how long it took you to solve each problem.

- 6-1, 10 pts. Practice with the interaction picture. Check that amplitudes in the Heisenberg picture and the Interaction picture are equivalent. Specifically, demonstrate that

$${}_H\langle\psi|\mathcal{O}_H(t)|\psi\rangle_H = {}_I\langle\psi(t)|\mathcal{O}_I(t)|\psi(t)\rangle_I , \quad (1)$$

where the left-hand side is written with operators and states in the Heisenberg picture and the right-hand side is written with operators and states in the Interaction picture. For convenience, you can choose $t_0 = 0$ (where t_0 is the reference time that relates the Interaction picture to the Heisenberg picture).

- 6-2, 25 pts. Practice with the unitary time evolution operator. We start with the unitary time evolution operator, defined with respect to the reference time t_0 .

$$U(t, t_0) \equiv T \left\{ \exp \left[-i \int_{t_0}^t dt' H_{\text{int}}^I(t') \right] \right\} . \quad (2)$$

We can remove the reference time t_0 and define

$$U(t_1, t_2) \equiv U(t_1, t_0) U^\dagger(t_2, t_0) . \quad (3)$$

A, 8 pts. Show that $U(t, t')$ obeys the same differential equation as $U(t, t_0)$.

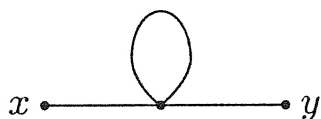
B, 8 pts. Show that $U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)$ (for $t_1 \geq t_2 \geq t_3$).

C, 9 pts. Show that

$$U(t, t') = T \left\{ \exp \left[-i \int_{t'}^t dt'' H_{\text{int}}^I(t'') \right] \right\} . \quad (4)$$

- 6-3, 25 pts. Prove Wick's theorem for an n -pt. correlation function by induction. In other words, assuming Wick's theorem holds for a correlation function of $n - 1$ real scalar fields, show that Wick's theorem is valid for a correlation function of n real scalar fields.
Hint: See page 90 of Peskin and Schroeder.

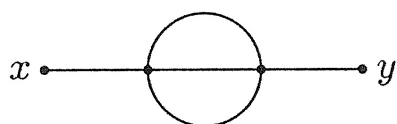
6-4, 40 pts. For each of the diagrams below (10 pts. each) from ϕ^4 theory, write a representative contraction of field operators (as in Equation 4.45 of Peskin and Schroeder). Moreover, calculate the multiplicity of the equivalent contractions for each diagram and check the listed symmetry factors. *Hint: The number of vertices will determine which power of λ is needed for the expansion of the exponential.*



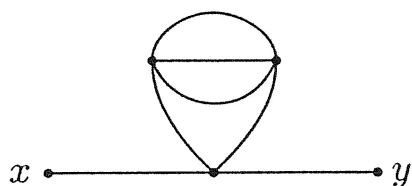
$$S = 2$$



$$S = 2 \cdot 2 \cdot 2 = 8$$



$$S = 3! = 6$$



$$S = 3! \cdot 2 = 12$$