Quantum Field Theory 1.

08.128.165 Theorie 6a, Relativistische Quantenfeldtheorie

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Lecture 1.

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- Administrivia - see syllabus
  - ★ Doodle poll to select time slot for HW discussion sessions
    - Start next week (week of April 19)
- Written exam - ~~sometime~~ July ~~xx~~ 23, 2021, 9am - 12pm, HS 19.
  - Must have 50% of HW credits to be eligible
- HW - generally due by email to yu001@uni-mainz.de by Monday lecture start at 10:00am
- Course assts: Julien Laux (jlaux01@uni)
  Alexey Kivel (alkivel@uni)
- All course materials, including lecture videos, posted on Moodle - HW 1 posted later today.

Introduction and Motivation

Why study & understand QFT?

① QFT is the standard framework for calculations in frontiers of physics, spanning condensed matter to particle physics.

For example, last week, April 7, 2021, the Fermilab E989 (g-2) Experiment published results of their first measurement of the anomalous magnetic moment of the muon. [Phys. Rev. D 103, 072002 (2021)], [PRL 126, 141801 (2021)]

The Fermilab result was

$$a_\mu = 116\,592\,040\,(54) \times 10^{-11} \qquad (0.46 \text{ ppm})$$

This is 20 years after an earlier result by the Brookhaven g-2 experiment, which gave

$$a_\mu = 116\,592\,092\,(63) \times 10^{-11}. \qquad [\text{PRD } 73, 072003 (2006)]$$

The Standard Model of particle physics gives a theoretical prediction of

$$a_\mu = 116\,591\,810\,(43) \times 10^{-11} \qquad [2006.04822]$$

QFT is the mathematical framework that underpins the theory calculation of muon g-2 and, in fact, all high & low energy particle physics data that we compare against experiment.

While there is significant excitement in the field from the current result (a combined 4.2σ discrepancy to the SM expectation), it does not discount the remarkable fact that QFT+SM predicts thousands of observables to extraordinary accuracy, which have been borne out by experimental measurements.

So one goal of these lectures is to develop an understanding of QFT that allows you to make physical predictions to compare with data.

In fact, we will make the one-loop calculation for g-2 of the electron in quantum electrodynamics at the end of the course.

(2) From a mathematical perspective, QFT is a very rich structure, combining quantum mechanics, special relativity + field theory [in physics language] or rather combining unitarity, Lorentz-invariance, and causality with field theory [in mathematics + physics language].

In this perspective, scattering amplitudes are characterized as functions of spacetime with a general structure of

$$S_{\alpha\beta} = \langle \alpha_{out} \mid S \mid \beta_{in} \rangle$$

out-state, distant future ↑ ↑ in-state, distant past

Here, S is the "S-matrix," commonly called the scattering matrix, with the closest analogy to interacting Hamiltonians in quantum mechanics where α + β are eigenstates.

In this course, we will refer to α + β as "asymptotic" states, which should be thought of as plane-wave, quantized solutions to a free Hamiltonian. IOW, can think α + β as populating the Hilbert space (technically, the Fock space defined by the direct sum of n-particle Hilbert spaces, which then allows for particle creation + annihilation).

Arbitrary mathematical functions of spacetime for S would violate unitarity + locality/causality, so generating a set of physical amplitudes is done using QFT.

Here "physical amplitude" is the crucial diffrence between mathematics + mathematical physics. Mathematical physics studies are restricted the mathematical functions that could be realized in Nature, or, in other versions of Nature if the underlying rules of Nature were changed (like how many dimensions we have, adding more spacetime symmetry like conformal or supersymmetry). The fact that the amplitudes calculated from QFT are physical gives great insight into the formal side of mathematics + also feeds into open, otherwise intractable problems in physics. Specifically, we can ~~realize~~ have many realizations of the same physical amplitude at long distances while the short distance amplitude behavior varies dramatically.

☆ This is the Wilsonian field theory point of view, where QFT is a framework that categorizes universality classes of physical theories that share the same long-range = low energy behavior.

☆ The viewpoint of QFT as a way to construct unitary + local / causal scattering amplitudes is known as the traditional view of QFT, attributed to Weinberg. These two viewpoints are compatible.

We will not get into the details of the Wilsonian picture until later in the course, when we introduce renormalization, since we need a notion of how a field theory is expressed at different energy scales.

Intuitive development of QFT.

Take the scattering amplitude as the starting point,

$$S_{\alpha\beta} \equiv \langle \alpha_{in} | S | \beta_{out} \rangle,$$

we will construct S as the unitary operator, Poincaré-invariant

$$S = T \exp\left(-i \int dt \int d^3x \, \mathcal{H}(\vec{x}, t)\right)$$

(T: time-~~ord~~ ordered product; $\mathcal{H}$: Hamiltonian density)

Given S is Poincaré invariant, we naturally assign α + β to ~~have well-defined~~ transform as irreps under Poincaré symmetry, so they are characterized by spin + energy-momentum (+ helicity). Moreover, these particles are reflected/realized as harmonic oscillator modes $a^\dagger_{\vec{p}}$, $a_{\vec{p}}$ on an underlying field (scalar, vector, fermion) that is defined over all $(\vec{x}, t)$. The quantization (and main distinction to classical field theory) is the imposition of non-vanishing commutation (for bosonic fields) or anti-commutation (for fermionic fields) relations on the raising + lowering operators.

"irrep" = irreducible representation

- We will see that only massless particles of spin $S \leq 2$ (half-integers from Lorentz invariance) can couple s.t. there are long-range forces.

Massless vector, spin-1: Must couple to conserved current j^μ.

Massless tensor, spin-2: Couples to $T^{\mu\nu}$, energy-momentum.

- Locality also constrains form of Hamiltonian.

$\Rightarrow$ $[\mathcal{H}(x), \mathcal{H}(y)] = 0$ at spacelike $(x-y)^2 < 0$
(Causality condition)

This is satisfied if we write theory in terms of a local Lagrangian (i.e. no higher derivative terms beyond kinetic terms).

- Great effort needed to impose & understand internal symmetries of S, $\Rightarrow$ gauge (dep. on x) or global (ind. of x)

Will need gauge conditions to remove gauge redundancies & retain manifest local, Poincaré invariance.

These sum up to the interpretation that to build a QFT description of Nature, we introduce one field for every type of particle (characterized by spin, gauge & quantum numbers) & excite particles by operating on the field via harmonic oscillator-type ladder operators. Particles can then be thought of as regular quantized wave-packets familiar from free field theory / QM.

Surprisingly (maybe?), can already exhaust all elementary particles with a very limited set of fields:

spin 0: Higgs
spin 1/2: quarks + leptons
spin 1: photon, $W^{\pm}$, Z, gluon
spin 2: graviton

These comprise SM & general relativity, our state-of-the art understanding of Nature.