08.128.165 Theorie 6a, Relativistische Quantenfeldtheorie Quantum Field Theory I

Felix Yu and Prisco Lo Chiatto, Martin Mojahed

Homework set 9

Due July 3, 2024 by start of lecture.
Please note how long it took you to solve each problem.

9-1, 25 pts. *Practice with Dirac algebra (part 4).* Verify the following identities. *Note: All of the following are written with u and v spinors, but they are generic for $u \leftrightarrow v$ interchange.*

- A, 10 pts. $(\bar{v}\gamma^\mu u)^\dagger = (\bar{u}\gamma^\mu v)$
- B, 10 pts. $(\bar{u}\gamma^\mu\gamma^5 v)^\dagger = (\bar{v}\gamma^\mu\gamma^5 u)$
- C, 5 pts. Given the answers to parts A and B, what is the Hermitian conjugate of $(\bar{v}\gamma^\mu P_L u)$ and $(\bar{v}\gamma^\mu P_R u)$?

9-2, 10 pts. Show that a given fermion field transforms the same as the covariant derivative of the field under a gauged $U(1)$ symmetry transformation.

9-3, 65 pts. *Bhabha scattering, Peskin and Schroeder problem 5.2.*

- A, 10 pts. Given the Feynman rules for QED, draw the two tree-level diagrams for $e^+e^- \to e^+e^-$ scattering, labeling all lines and vertices as necessary.
- B, 10 pts. Write the two matrix elements for these diagrams. What is the relative sign between the two diagrams?
- C, 20 pts. Setting the electron mass to 0, solve for the differential cross section $d\sigma/d\cos\theta$ for Bhabha scattering. *Hint: Recall that we have identities for Lorentz-contracted γ matrices surrounding other gamma matrices, from HW problem 4-4.* Reexpress the kinematic invariants the Mandelstam variables, and note that for massless electrons, $s + t + u = 0$. You should get the result

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left[u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2\right] \qquad (1)$$

- D, 10 pts. Rewrite the differential cross section in terms of s and the scattering angle $\cos\theta$. What is the leading behavior as $\theta \to 0$, and what feature of the diagrams causes this behavior?
- E, 15 pts. For the total cross section, we should impose a cut on $\theta > \theta_0$, with $\theta_0 \ll 1$, since it is impossible to distinguish forward scattering from no scattering at all (in contrast to $e^+e^- \to \mu^+\mu^-$). Integrate the differential cross section with such a cutoff, keeping only terms that are singular or constant in the limit $\theta_0 \to 0$.