08.128.165 Theorie 6a, Relativistische Quantenfeldtheorie Quantum Field Theory I

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Homework set 6

Due June 12, 2024 by start of lecture.
Please note how long it took you to solve each problem.

6-1, 10 pts. Practice with the interaction picture. Demonstrate that

$$_H\langle\psi|\mathcal{O}_H(t)|\psi\rangle_H = {}_I\langle\psi(t)|\mathcal{O}_I(t)|\psi(t)\rangle_I \ , \tag{1}$$

where the left-hand side is written with operators and states in the Heisenberg picture and the right-hand side is written with operators and states in the Interaction picture. For convenience, you can choose $t_0 = 0$, where t_0 is the reference time that relates the Interaction picture to the Heisenberg picture, meaning $\mathcal{O}_H(t = 0) = \mathcal{O}_I(t = 0)$.

6-2, 25 pts. Practice with the unitary time evolution operator. We start with the unitary time evolution operator, defined with respect to the reference time t_0.

$$U(t, t_0) \equiv T\left\{\exp\left[-i\int_{t_0}^{t} dt' H^I_{\text{int}}(t')\right]\right\} \ . \tag{2}$$

We can remove the reference time t_0 and define

$$U(t_1, t_2) \equiv U(t_1, t_0)U^\dagger(t_2, t_0) \ . \tag{3}$$

A, 8 pts. Show that $U(t, t')$ obeys the same differential equation as $U(t, t_0)$.

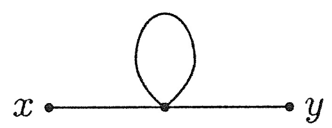
B, 8 pts. Show that $U(t_1, t_2)U(t_2, t_3) = U(t_1, t_3)$ (for $t_1 \geq t_2 \geq t_3$).

C, 9 pts. Show that

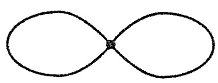
$$U(t, t') = T\left\{\exp\left[-i\int_{t'}^{t} dt'' H^I_{\text{int}}(t'')\right]\right\} \ . \tag{4}$$

6-3, 25 pts. Prove Wick's theorem for an n-pt. correlation function by induction. In other words, assuming Wick's theorem holds for a correlation function of $n - 1$ real scalar fields, show that Wick's theorem is valid for a correlation function of n real scalar fields. *Hint: See page 90 of Peskin and Schroeder.*

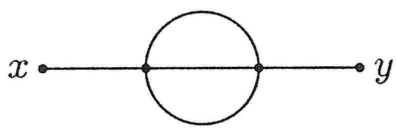
6-4, 40 pts. For each of the diagrams below (10 pts. each) from ϕ^4 theory, write a representative contraction of field operators (as in Equation 4.45 of Peskin and Schroeder). Moreover, calculate the multiplicity of the equivalent contractions for each diagram and check the listed symmetry factors. *Hint: The number of vertices will determine which power of λ is needed for the expansion of the exponential.*



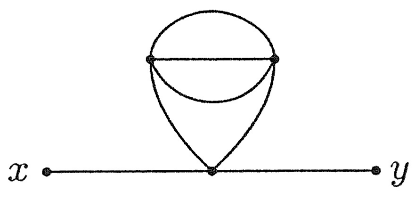
$S = 2$



$S = 2 \cdot 2 \cdot 2 = 8$



$S = 3! = 6$



$S = 3! \cdot 2 = 12$