08.128.165 Theorie 6a, Relativistische Quantenfeldtheorie Quantum Field Theory I

Felix Yu and Alexey Kivel, Julien Laux

Formula sheet for the final exam.

Noether current

Given a Lagrangian invariant under a continuous transformation law of the dynamical fields,

$$\phi(x) \to \phi'(x) = \phi(x) + \alpha \Delta\phi(x) \ , \tag{1}$$

the Noether current $j^\mu(x)$ is a conserved quantity,

$$\partial_\mu j^\mu(x) = 0 \ , \tag{2}$$

with

$$j^\mu(x) = \sum_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \Delta\phi_i \ . \tag{3}$$

Here, we have assumed the Lagrangian itself is invariant under the transformation parametrized by α.

Gauge condition

In this course, we generally use the Lorenz gauge condition for quantum electrodynamics. That is, we require photon fields obey the gauge condition

$$\partial_\mu A^\mu(x) = 0 \ . \tag{4}$$

For on-shell photons, where the photon is an external particle in a cross section or decay width calculation, the gauge condition requires that

$$k_\mu \epsilon^\mu(k) = 0 \ , \tag{5}$$

where k is the 4-momentum of the photon with polarization vector $\epsilon^\mu(k)$.

Cross sections

The differential cross section for two particles scattering into $\{f\}$ final state particles is

$$d\sigma = \frac{1}{2E_A 2E_B |v_A - v_B|} \left(\Pi \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \right) \tag{6}$$

$$\times |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)} \left(p_A + p_B - \sum p_f \right) \ . \tag{7}$$

For the special case of two particles in the final state, we can write the two-body Lorentz-invariant phase space in the center-of-mass frame as

$$\int d\Pi_2 = \int d\Omega \frac{1}{16\pi^2} \frac{|\vec{p}_1|}{E_{\rm cm}} \ , \tag{8}$$

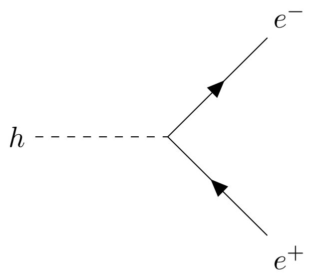
where $E_{\rm cm}$ is the total energy in the center of mass frame and $|\vec{p}_1| = |\vec{p}_2|$ is the magnitude of the three-momentum of one of the outgoing particles.

In the special case where all four particles in two-to-two scattering have identical masses, the differential cross section can be written in the CM frame as

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm cm} = \frac{|\mathcal{M}|^2}{64\pi^2 E_{\rm cm}^2} \ . \tag{9}$$

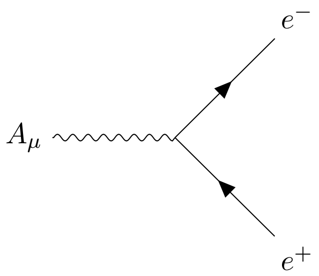
Interaction Feynman rules

Yukawa coupling



$$= -i\frac{y}{\sqrt{2}}$$

QED coupling



$$= ieQ\gamma^\mu$$