Office hours, May 26, 2021

Wednesday, May 26, 2021 2:11 PM

General questions? Specific questions? Technical questions?

6-1. Hermeticity of fermion bilinears.

Check $(\bar{\Psi}\psi)^\dagger = \bar{\Psi}\psi$

LHS: $(\bar{\Psi}\psi)^\dagger = (\psi^\dagger\gamma^0\psi)^\dagger$

$$= \psi^\dagger (\gamma^0)^\dagger (\psi^\dagger)^\dagger = \psi^\dagger \gamma^0 \psi = \bar{\Psi}\psi$$

Think $\bar{\Psi}$: row in spin space ; ψ : column in spin space

$(\xi^T \; \eta^T)$ 4-comp. row

$\begin{pmatrix} \xi \\ \eta \end{pmatrix}$: 4-comp. column

$\bar{\Psi}\psi$ = scalar

$(\bar{\Psi}\psi)^\dagger = (\text{scalar})^\dagger$

$(\text{row} \cdot \text{column})^T = (\text{column})^T (\text{row})^T = \text{row} \cdot \text{column}$

Bilinear:

Linear: polynomial in one variable: $f(x) = a + \underbrace{bx} + cx^2 + \dots$

$g(y) = \alpha + \beta y + \gamma y^2 + \dots$

$\Rightarrow$ $h(x,y) \overset{\text{relabel}}{=} A + Bx + B'y + Cx^2 + C'xy + C''y^2 + Dx^3 + D'x^2y + D''xy^2 + D'''y^3 + \dots$

$= \underline{f(x)\, g(y)}$

$(a + bx + cx^2 + \dots)(\dots)$

$$- \overline{f(x)\, g(y)}$$

$$(a + bx + cx^2 + \dots)\ (\alpha + \beta y + \gamma y^2 + \dots)$$

$$= a\alpha + \alpha b x + a\beta y + \alpha c x^2 + \underbrace{b\beta xy}_{\text{bilinear}} + a\gamma y^2 + \dots$$

$f(x)$, $g(y)$: polynomials of one variable

polynomial of two variables?

$$h(x,y) = \underbrace{f(x) \circ g(y)}\quad \text{vs.}\quad h(xy) \leftarrow \text{less interesting}$$

$$f(x) \circ f(y) \leftarrow h(xy) \quad \sum_{n=0}^{\infty \text{ all}} \frac{(xy)^n}{n!} h^{(n)}(x_0 y_0)$$

This also defines/explains how a tensor is different than a matrix.

Case B: Solve polynomial : solve roots of polynomial

Map: coefficients to vector

Case A: Solving linear systems of equations:

$$\begin{cases} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 = b_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 = b_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 = b_3 \end{cases}$$

$$\begin{pmatrix} & A & \end{pmatrix} \begin{pmatrix} x \end{pmatrix} = \begin{pmatrix} b \end{pmatrix}$$

Eliminate variables, solve $\{x_1, x_2, x_3\}$ by RREF.

Linear system of equations: $\{x_1, x_2, x_3\}$ solution.

Now, $$a x_1 x_2 + b x_1 y_2 + c x_2 y_1 + d y_1 y_2 = \alpha$$
$$e x_1 x_2 + f x_1 y_2 + c x_2 y_1 + h y_1 y_2 = \beta$$

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix} \underbrace{\begin{bmatrix} x_1 x_2 \\ x_1 y_2 \\ y_1 x_2 \\ y_1 y_2 \end{bmatrix}}_{\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \begin{bmatrix} x_2 & y_2 \end{bmatrix}} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

$$M_1 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \cdot M_2 \begin{bmatrix} x_2 & y_2 \end{bmatrix} \quad \text{s.t.} \quad M \begin{bmatrix} x_1 x_2 \\ x_1 y_2 \\ y_1 x_2 \\ y_1 y_2 \end{bmatrix}$$

$$f(x) \circ g(y) = h(x, y)$$

composition fcn. $\Rightarrow$

Define tensor: $$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \otimes \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \equiv \begin{bmatrix} x_1 x_2 & x_1 y_2 \\ y_1 x_2 & y_1 y_2 \end{bmatrix}$$

(dyadic product)

Composition of two column vectors.

rescale $$\begin{bmatrix} 2x_1 \\ \cancel{2} y_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 2x_1 x_2 & 2x_1 y_2 \\ \cancel{2} y_1 x_2 & \cancel{2} y_1 y_2 \end{bmatrix}$$

vs

$$\begin{bmatrix} y_2 \end{bmatrix} = \begin{bmatrix} \cancel{2} y_1 x_2 & \cancel{2} y_1 y_2 \end{bmatrix}$$

vs

$$\begin{bmatrix} 2x_1 \\ y_1 \end{bmatrix} \cdot \begin{bmatrix} x_2 \\ \frac{1}{2} y_2 \end{bmatrix} = \begin{bmatrix} 2x_1 x_2 & x_1 y_2 \\ y_1 x_2 & \frac{1}{2} y_1 y_2 \end{bmatrix}$$

Metric tensor:

Lorentz transformation $\Lambda^\mu{}_\nu$ acting on x^ν

$$x^\mu \xrightarrow{LT} x'^\mu = \underbrace{\Lambda^\mu{}_\nu}\, x^\nu$$

$$h^{\mu\nu} \xrightarrow{L.T.} h'^{\mu\nu} = \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta\, h^{\alpha\beta}$$

$h^{\alpha\beta}$ is composition of two four-vectors, like $j^\alpha k^\beta$.

freeze α, then $h^{[\alpha]\beta}$ is four-vector

freeze β, then $h^{\alpha[\beta]}$ is a four-vector.

Define: $j^\alpha k^\beta \equiv h^{\alpha\beta}$

$$\begin{bmatrix} \text{4-} \\ \text{vec.} \end{bmatrix} \begin{bmatrix} \text{4-} \\ \text{vec.} \end{bmatrix} = \begin{bmatrix} h^{\alpha\beta} \end{bmatrix} \Rightarrow$$

outer product vs. inner product

$g_{\alpha\beta}\, j^\alpha k^\beta = j_\beta k^\beta =$

<u>not</u> composition, contracting k^β space w/ dual space

Γ^A, Γ_B basis of Dirac matrices:

Γ^A act on Ψ

Γ_B acting on $\bar{\Psi}$ dual to Γ^A

Conceptual diff. b/t QM & QFT.

In QM, probabilistic interpretation ... exp. value of

In QM, probabilistic interpretation of operators (such as $\langle x \rangle$ exp. value of)

In QFT: exp. value:

Since fields are composed of ladder operators, consider vacuum expectation values = value of field sandwiched in vacuum.

In relativistic QFT, $\langle \Omega | \Psi | \Omega \rangle \neq 0$ or $\langle \Omega | A_\mu | \Omega \rangle \neq 0$

interacting vacuum

then Lorentz Sym. is broken.

In condensed matter QFT, you may want: preferred spin axis or polarization for dof.

The only possibility for us is $\langle \Omega | \phi | \Omega \rangle \neq 0$ & keeps Lorentz symmetry. [Also $\langle \Omega | \bar{\Psi} \Psi | \Omega \rangle \neq 0$]

Realized in Nature:

$$\langle \Omega | H | \Omega \rangle = v = 246 \text{ GeV}$$

QCD condensate $\langle \Omega | \bar{q} q | \Omega \rangle \approx 200 \text{ MeV} = \Lambda_{QCD}$

Otherwise, this is amplitude for scattering.

$\langle \phi | O | \bar{\Psi} \Psi \rangle$ = "expectation value of O with initial state $\bar{\Psi}\Psi$ & final state ϕ"

bra ket

$\langle \phi | O | \phi \rangle$ = " ... initial ϕ & final ϕ"

$\parallel$

$\langle \Omega | \phi \, O \, \phi | \Omega \rangle =$

$\langle \Omega | \underbrace{\phi \, \mathcal{O} \, \phi} | \Omega \rangle =$

ϕ^5 , $\phi A_\mu A^\mu \phi$, $\phi \bar{\Psi} \Psi \phi^7$

corr. fcn. for these particles to pop out of vacuum + scatter = amplitude for $\langle f | S | i \rangle$.

$\mathcal{L} \supset z_\mu A^\mu + \ldots$

z = fixed $\qquad \langle \Omega | \; z_\mu A^\mu | \Omega \rangle \propto \sqrt{z^2} \neq 0$

---

time evolution operator

$$U(t, t_0) = 1 - i \int_{t_0}^{t} dt_1' \underbrace{H_{int}^{I}}_{\frac{\lambda_3 \phi^3}{3!}} (t_1') + \ldots$$

$U(t, t_0) - 1$ = H_{int} is responsible for the difference b/t Heisenberg + Interaction picture.

Leading term: $-i \int_{t_0}^{t} dt_1 \frac{\lambda_3 \phi^3}{3!} \approx -i \frac{\lambda_3 \cdot \phi^3}{3!} \Delta t$

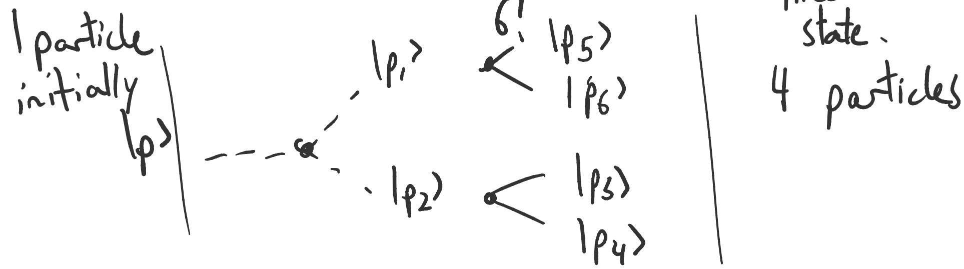
Scaling is alway $\lambda_3 \Delta t$

this is the connection to adiabatic transitions.

$$\phi_A(x) = U^\dagger(t, t_0) \, \phi_I(t, \vec{x}) \, U(t, t_0)$$

We take λ_3 tiny, + Δt long, but essentially want to control the overall scaling $\lambda_3 \Delta t$ (+ approximate $\phi^3 \sim \phi_{free}^3$ with fixed time dependence)

Use $H_{int} \sim \frac{\lambda_3 \phi^3}{6!}$



Away from interactions, particles are treated purely as <u>free states</u>.