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Quantum Field Theory.

Note on Multiparticle states and Hilbert + Fock Space.

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(based on Neubert Lecture 4 notes)

Multiparticle states.

Operate ladder operators on vacuum $|0\rangle$.

Ex: $a^\dagger_{\vec{p}}|0\rangle, \quad a^\dagger_{\vec{p}_1} a^\dagger_{\vec{p}_2}|0\rangle, \ldots$

These are particles in momentum space,

$$|\vec{p}_1 \, \vec{p}_2 \ldots \vec{p}_N\rangle = a^\dagger_{p_1} a^\dagger_{p_2} \ldots a^\dagger_{p_N} |0\rangle$$

Since $a^\dagger$ commute with each other, particles are <u>bosonic</u> & they are <u>bosons</u>.

For higher occupancy numbers, we symmetrize by factorials:

$$|\underbrace{\vec{p}_1 \ldots \vec{p}_1}_{n_1} \, \underbrace{\vec{p}_2 \ldots \vec{p}_2}_{n_2} \ldots \underbrace{\vec{p}_N \ldots \vec{p}_N}_{n_N}\rangle = \prod_{i=1}^{N} \frac{1}{\sqrt{n_i!}} \underbrace{a^\dagger_{p_i} \ldots a^\dagger_{p_i}}_{n_i} |0\rangle$$

Multiparticle states can also be written as a tensor product of one-particle states:

$$\ldots = \frac{1}{\sqrt{N!}} \left(\prod_i \frac{1}{\sqrt{n_i!}}\right) \sum_{\sigma \in S_N} P_\sigma \left(|\vec{p}_1\rangle \otimes \ldots \otimes |\vec{p}_i\rangle \otimes \ldots \otimes |\vec{p}_N\rangle \otimes \ldots \otimes |\vec{p}_N\rangle\right)$$

Fock space

$$F_\nu = \bigoplus_{n=0}^{\infty} S_\nu H^{\otimes n} \quad ; \quad H^{\otimes n} = H \otimes \ldots \otimes H$$

single particle Hilbert space

(anti)symmetrization operator for bosons (fermions)

$H^{\otimes 0} \equiv |0\rangle$ vacuum

Number operator $N = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_{\vec{p}}} n_{\vec{p}} = \int \frac{d^3p}{(2\pi)^3} \frac{a^\dagger_{\vec{p}} a_{\vec{p}}}{2E_{\vec{p}}}$

satisfies $N|\vec{p}_1 \ldots \vec{p}_N\rangle = n |\vec{p}_1 \ldots \vec{p}_N\rangle$. Commutes with $[H_{free}, N] = 0$, so particle number is conserved.