Quantum Field Theory I.

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Quantum Mechanics Primer

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Starting with Schrödinger's equation,

$$i\hbar \frac{\partial}{\partial t}\Psi = \left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)\right]\Psi(x,t)$$

we want to understand general ways to classify solutions for $\Psi(x,t)$. The key complication from quantum mechanics is that $[\hat{x},\hat{p}] = i\hbar$, (Heisenberg uncertainty principle), which prevents us from knowing the state of $\Psi(x,t)$ and $\tilde{\Psi}(p,t)$ simultaneously. (Fourier transform) At best, we can saturate the uncertainty relation by wave packets, which propagate with group + phase velocity matched (without dissipation).

The non-commutation of operators also naturally leads us to use eigentheory to best simplify and make practical progress in calculations. From this perspective, the difficulties of quantum mechanics become encoded by incompatible choices of basis functions (such as position vs. momentum eigenstates). If the form of the Hamiltonian is particularly simple, then we can actually find sets of eigenfunctions that are commensurate. Example: Harmonic oscillator. H commutes with number operator + has simple commutation relations with ladder operators.

The power of commuting operators is this: while we can always assume an eigenbasis and assign eigenvalues for a given (solvable) operator like $\frac{p^2}{2m}$, the ~~commutation~~ commutivity with another operator means we can ~~reu~~ reuse the same basis for the second operator. If we include time evolution & if the initial eigenstate is an eigenstate of the full Hamiltonian, then the eigenstate only evolves with a phase, never "rotating" to a different eigenstate. We can, by orthogonality of eigenbases, assign eigenvalues as <u>conserved quantum numbers</u>, when the operator in question commutes with H. The key is that we can <u>factorize</u> the Hilbert (or Fock space to account for multiparticle wavefcns) by these eigenvalue / quantum number choices, since the Hamiltonian ~~with only~~ conserves the assigned quantum number.

As a final remark, we also can understand the composition of total angular momentum into orbital and spin angular momentum. Without $\vec{L}$ or $\vec{S}$ operators, a state with only $\vec{J}$ operators acting on it necessarily only has total J and J_z eigenvalues. But once we include more structure (new operators in the Hamiltonian that have diff. symmetries or quantum #s compared to ~~ex~~ existing terms), then there are new relations that constrain the subspaces that $\vec{L}$ operates on compared to $\vec{S}$ operations, given by $\vec{J} = \vec{L} + \vec{S}$, etc.