Lecture 11.
Quantum Field Theory 1.
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Last time: Free field theory of fermions.
Today: Discrete symmetries of Lorentz symmetry.

We have 3 possible discrete symmetries, which are especially relevant or non-trivial now that we have discussed spin ½ fields.

They are C = charge conjugation.
P = parity transformation.
T = time reversal.

Note that P + T are implemented as discrete transformations in the Lorentz group.

$$P: (t, \vec{x}) \to (t, -\vec{x}).$$
$$T: (t, \vec{x}) \to (-t, \vec{x}).$$

Importantly, these discrete transformations preserve the four-vector scalar product distance, $s^2 = x^\mu x_\mu = t^2 - |\vec{x}|^2$.
On the other hand, they are implemented as a Lorentz transformation with $\det = -1$ (acting on 4-vectors):

$$\Lambda_{P}{}^{\mu}{}_{\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad \Lambda_{T}{}^{\mu}{}_{\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

Thus, they are not smoothly deformable from the identity Lorentz transformation. (with $\det = 1$).
The continuous generators w/o P + T are known as the proper, orthochronous Lorentz group.

The continuous Lorentz generators with P = "improper, orthchronous"
Continuous Lorentz with T = "proper, non-orthochronous"
Continuous Lorentz with $P + T$ = "improper, non-orthochronous".

In Nature, we have 4 fundamental forces. Of these, gravity, electromagnetism, and the strong interactions behave the same under particles or antiparticles + their parity-partners, as well as whether time is normal or reversed. (Here, we mean that the dynamical equations are invariant under $t \to -t$.) The weak interactions, however, explicitly violate C and P: interactions for a given particle do not exist for the charge conjugate or the parity partner, but CP + T are preserved (by the EOMs). We have also experimentally verified CP and T violation in physical processes such as kaon oscillations, B-meson decays, + D-meson decays. Moreover, we have no experimental evidence for CPT violation, so we expect that CPT is a good symmetry of Nature.

How do $C, P + T$ act on Dirac fermions?

For P, we want

$$P a^s_{\vec{p}} P = \eta_a a^s_{-\vec{p}} , \qquad P b^s_{\vec{p}} P = \eta_b b^s_{\vec{p}}$$

where η_a, η_b are phases, requiring $\eta_a^2 = \pm 1$, $\eta_b^2 = \pm 1$.
Can implement P as constant 4×4 matrix, and can find from $P \Psi(x) P$ = something $\cdot \Psi(t, -\vec{x})$ that we need $\eta_b^* = -\eta_a$ + $P\Psi(x)P = \eta_a \gamma^0 \Psi(t, -\vec{x})$

Hermitian bilinears:

$$\bar\psi\psi,\quad \bar\psi\gamma^\mu\psi,\quad i\bar\psi[\gamma^\mu,\gamma^\nu]\psi,\quad \bar\psi\gamma^\mu\gamma^5\psi,\quad i\bar\psi\gamma^5\psi.$$

Can verify:
$$P\bar\psi(t,\vec{x})P = P\psi^\dagger(t,\vec{x})P\gamma^0$$
$$= (P\psi(t,\vec{x})P)^\dagger\gamma^0$$
$$= \eta_a^*\,\bar\psi(t,-\vec{x})\gamma^0$$

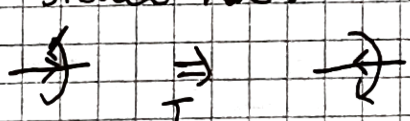
and $P\bar\psi\psi P = +\bar\psi\psi(t,-x)$

[Will generally not care about η_a or η_b because observables are always bilinears of fermions, so we can set $\eta_a = -\eta_b = 1$ from beginning.]

Can also check:
$$P\, i\bar\psi\gamma^5\psi P = -i\bar\psi\gamma^5\psi(t,-\vec{x})$$
$$P\bar\psi\gamma^\mu\psi P = (-1)^\mu\bar\psi\gamma^\mu\psi(t,-\vec{x})$$
$$P\bar\psi\gamma^\mu\gamma^5\psi P = -(-1)^\mu\bar\psi\gamma^\mu\gamma^5\psi(t,-\vec{x})$$
$$P\bar\psi\sigma^{\mu\nu}\psi P = (-1)^\mu(-1)^\nu\bar\psi\sigma^{\mu\nu}\psi(t,-\vec{x})$$
for $(-1)^\mu \equiv (1,-1,-1,-1)$

Time reversal: Must be implemented as antiunitary operator:
$$T(\text{c-number}) = (\text{c-number})^*T$$
$$\Rightarrow\ Te^{+iHt} = e^{iH(-t)}T,$$ effectively changes sign of t.

T should reverse momentum & flip spin.



Details in Section 3.6 of P+S.

Argument: construct explicit transformation on $\xi^s \to \xi^{-s}$ and $u^s(p) \to u^{-s}(\tilde{p})$, get $Ta^s_{\vec p}T = a^{-s}_{-\vec p}$, $Tb^s_{\vec p}T = b^{-s}_{-\vec p}$

and $T\psi(t,\vec{x})T = \gamma^1\gamma^3\psi(-t,\vec{x})$, $T\bar\psi T = \bar\psi(-t,\vec{x})(-\gamma^1\gamma^3)$

Other bilinears:

$$T\, i\bar{\Psi}\gamma^5\Psi = -i\,\bar{\Psi}\gamma^5\Psi(-t,\vec{x})$$
$$T\bar{\Psi}\gamma^\mu\Psi T = (-1)^\mu\,\bar{\Psi}\gamma^\mu\Psi(-t,\vec{x})$$
$$T\bar{\Psi}\gamma^\mu\gamma^5\Psi T = (-1)^\mu\,\bar{\Psi}\gamma^\mu\gamma^5\Psi(-t,\vec{x})$$
$$T\bar{\Psi}\sigma^{\mu\nu}\Psi T = -(-1)^\mu(-1)^\nu\,\bar{\Psi}\sigma^{\mu\nu}\Psi(-t,\vec{x})$$

Finally: charge conjugation

Keep spin & momentum & mass (quantum numbers of spacetime), change internal quantum numbers, like charge.

$$C a^s_{\vec{p}} C = b^s_{\vec{p}} \quad ; \quad C b^s_{\vec{p}} C = a^s_{\vec{p}}$$

We get $u^s(p) = -i\gamma^2(v^s(p))^*$, $v^s(p) = -i\gamma^2(u^s(p))^*$

$$C\Psi(x)C = -i\gamma^2\Psi^*(x) = -i(\bar{\Psi}\gamma^0\gamma^2)^T$$

$$C\bar{\Psi}(x)C = (-i\gamma^2\Psi)^T\gamma^0 = (-i\gamma^0\gamma^2\Psi)^T$$

Then, on bilinears,

$$C\bar{\Psi}\Psi C = \bar{\Psi}\Psi$$
$$C\bar{\Psi}\, i\gamma^5\Psi C = i\bar{\Psi}\gamma^5\Psi$$
$$C\bar{\Psi}\gamma^\mu\Psi C = -\bar{\Psi}\gamma^\mu\Psi$$
$$C\bar{\Psi}\gamma^\mu\gamma^5\Psi C = \bar{\Psi}\gamma^\mu\gamma^5\Psi$$
$$C\bar{\Psi}\sigma^{\mu\nu}\Psi C = -\bar{\Psi}\sigma^{\mu\nu}\Psi$$

Under, P, T, C, we ~~too~~ also have

$$P\partial_\mu P = (-1)^\mu\partial_\mu \quad \& \quad T(\partial_\mu)T = -(-1)^\mu\partial_\mu, \quad C\partial_\mu C = \partial_\mu.$$

Since $\bar{\Psi}\Psi$, $i\bar{\Psi}\gamma^5\Psi$ under CPT is +1, any term in a Lagrangian must be invariant under CPT, if the Lagrangian is local & Lorentz-invariant.

Physics of C, P, T:

- Historically, since E+M and gravity obeyed parity symmetry, there was an assumption that the weak interactions (responsible for nuclear decay processes) were also parity. C.S. Wu experiment (1956) demonstrated that Co^{60} maximally violated parity in decays. Today, we understand it by attributing weak interactions only to LH particles and not RH particles. This has additional implications on fermion mass generation & the necessity of the Higgs mechanism for spontaneous symmetry breaking.
- Kaon decays: The existence of two different decay channels for $K^+ \to \pi^0 \pi^+$ & $K^+ \to \pi^+ \pi^- \pi^+$ also demonstrated parity violation. The final states, since pions are pseudoscalar fermion bilinears, are opposite parity, and so a single particle decaying to both states implies that parity is <u>not</u> a good selection rule in weak interactions.
- C-violation: Since C does not change spin, C is also violated by weak interactions maximally. LH ν acts, but LH $\bar{\nu}$ does not. (H.c. is RH $\bar{\nu}$).
- CP is also violated in SM: baryogenesis puzzle given
  - Particles & antiparticles can decay differently to final states.
- CPT is preserved, as far as we know.
  - particles + antiparticles have same masses.
- Majorana fermions: $C\Psi^* C = \Psi$. (Like real scalars.)