Lecture 8. ①

18.12.18
Special Makeup Lecture.

SM anomaly cancellation + tips for calculating + cancelling anomalies.

SM fermion content: Gauge reps. under $SU(3)\times SU(2)\times U(1)$

$$Q_L \sim (3, 2, 1/6)$$
$$u_R \sim (3, 1, 2/3)$$
$$d_R \sim (3, 1, -1/3)$$
$$L_L \sim (1, 2, -1/2)$$
$$e_R \sim (1, 1, -1)$$

Gauge anomalies cancel with each generation.
Must check for each gauge group at each vertex of triangle diagram.
Also, there are gravitational anomalies:

L. Alvarez-Gaumé + E. Witten, Nucl. Phys. B 234, 269 (1984)

Anomalies to check:

$SU(3)^3$, $SU(3)^2\times SU(2)$, $SU(3)^2\times U(1)_Y$,
$SU(2)^3$, $SU(2)^2\times SU(3)$, $SU(2)^2\times U(1)_Y$,
$U(1)^3$, $U(1)^2\times SU(3)$, $U(1)^2\times SU(2)$,
$SU(3)\times SU(2)\times U(1)_Y$
$SU(3)\times G^2$, $SU(2)\times G^2$, $U(1)\times G^2$.

Recall, $\sum_f A = 0$ consistency condition for Ward identity.

$$A = \mathrm{Tr}\left[t^a,\{t^b,t^c\}\right] A^{ABJ}$$

Exercise: Calculate explicitly!

Tips + tricks: ②

① In mixed anomalies, trace vanishes if single $SU(N)$ generator is present, since $SU(N)$ generators are traceless.
So: $SU(3)^2 \times SU(2)$, $SU(2)^2 \times SU(3)$, $U(1)^2 \times SU(3)$, $U(1)^2 \times SU(2)$, $SU(3) \times SU(2) \times U(1)$, $SU(3) \times G^2$, $SU(2) \times G^2$ all automatically vanish, fermion by fermion.

② $SU(2)^3$ always vanishes, since

$$\mathrm{Tr}\left[\tau^a \{\tau^b, \tau^c\}\right] = \frac{1}{8}\mathrm{tr}\left[\sigma^a \{\sigma^b, \sigma^c\}\right] = \frac{1}{8}\mathrm{tr}\left[\sigma^a\, 2\delta^{bc}\right] = 0$$

For general $SU(N)$, the anticommutator includes a (usually) nontrivial d^{abc}: $\{t^a, t^b\} = \frac{1}{d}\delta^{ab}\mathbb{1}_d + d^{abc}t^c$

d = dim of irrep.

In $SU(2)$, $d^{abc} = 0$.

③ Fermions are vector-like under QCD. In other words, if QCD were the only gauge group, we could write down explicit Dirac masses for all fermions. (In fact, after EWSB, $SU(3)_c \times U(1)_{em}$ gauge groups are pure vector gauge groups, which is why we can have massive fermions + unbroken QCD + EM in the first place.) Explicitly, if we only focus on $SU(3)$, we have:

$$\begin{matrix} u_L \sim 3 & u_R \sim 3 \\ d_L \sim 3 & d_R \sim 3 \end{matrix} \Rightarrow \text{allows} \quad m_u (\bar{u})(P_L + P_R)u + m_d (\bar{d})(P_L + P_R)d$$

(or even $M^{ij}\, \bar{q}_i q_j$ for $q = \begin{pmatrix} u \\ d \end{pmatrix}$ in flavor space)

Thus, treating <u>RH fields as charge conjugate LH fields</u>, we conclude u_L cancels u_R + d_L cancels d_R in $SU(3)^3$.

★ Easiest to always write all fields as LH.

Remaining anomalies to check: Ignore overall # of generations factor.

$SU(3)^2 \times U(1)_Y$:

$$\underset{\substack{\uparrow \\ Q_L:\ SU(2) \\ \text{multiplicity}}}{2} \times \mathrm{tr}\left[\tfrac{1}{2}\{t^b,t^c\}\right]\cdot\frac{1}{6} + \underset{u_R^c:}{\mathrm{tr}[\ldots]}\cdot\underset{\substack{\uparrow \\ \text{charge} \\ \text{conj.}}}{\left(\frac{-2}{3}\right)} + \underset{d_R^c}{\mathrm{tr}[\ldots]}\cdot\underset{\substack{\uparrow \\ \text{charge} \\ \text{conj.}}}{\left(\frac{+1}{3}\right)} \boxed{=0}$$

$SU(2)^2 \times U(1)_Y$:

$$Q_L \rightarrow \underset{\substack{\uparrow \\ SU(3) \\ \text{multiplicity}}}{3} \cdot \mathrm{tr}\left[\mathbb{1}\{\tau^b,\tau^c\}\right]\cdot\frac{1}{6} + \underset{L_L}{\mathrm{tr}\left[\mathbb{1}\{\tau^b,\tau^c\}\right]}\cdot\frac{-1}{2} \boxed{=0}$$

$U(1) \times G^2$: Sum over all $U(1)$ charges.

$$\underset{\substack{\uparrow \\ Q_L}}{3\cdot 2\cdot\frac{1}{6}} + \underset{\substack{\uparrow \\ u_R^c}}{3\cdot\frac{-2}{3}} + \underset{\substack{\uparrow \\ d_R^c}}{3\cdot\frac{+1}{3}} + \underset{\substack{\uparrow \\ L_L}}{2\cdot\frac{-1}{2}} + \underset{\substack{\uparrow \\ e_R^c}}{1\cdot(+1)} \boxed{=0}$$

$U(1)^3$: Related to Diophantine eqns.

$$3\cdot 2\left(\frac{1}{6}\right)^3 + 3\left(\frac{-2}{3}\right)^3 + 3\left(\frac{1}{3}\right)^3 + 2\left(\frac{-1}{2}\right)^3 + 1(+1)^3$$

$$= \frac{1}{36} - \frac{8}{9} + \frac{1}{9} - \frac{1}{4} + 1 = \frac{1}{36} + \frac{8}{36} - \frac{9}{36} \boxed{=0}$$

Discussion:

(A) The fact that anomalies vanish generation by generation does not fix the number of generations in SM to 3. Having 3 generations in SM is experimentally tested (from non-decoupling in Higgs physics).

Note that the # of quarks & charged leptons must match though. Contrast to the question of how many RH neutrinos we want ⇐ no role in anomaly cancellation, so theorist's choice.

B Gauge symmetries cannot be anomalous + be consistent in the UV, but global symmetries can be anomalous.

Recall: global symmetry of free SM (no interactions)

is $U(3)^5 = U(3)_Q \times U(3)_u \times U(3)_d \times U(3)_L \times U(3)_{e_R}$

When we include Yukawas, breaks to $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$

When we include RH neutrinos + Dirac masses, becomes $U(1)_B \times U(1)_L$.

These global symmetries are each anomalous since SM EW group is chiral. Quarks have baryon charge $\frac{1}{3}$, leptons have L-charge $+1$.

In particular, $\mathcal{A}(SU(2)^2 \times U(1)_B) = \mathcal{A}(SU(2)^2 \times U(1)_L) = \frac{3}{2}$

Ref. Perez, Wise, [1002.1754] $\mathcal{A}(U(1)_Y^2 \times U(1)_B) = \mathcal{A}(U(1)_Y^2 \times U(1)_L) = -\frac{3}{2}$

So, $U(1)_{B-L}$ is anomaly-free, but other linear combinations of $U(1)_B$ + $U(1)_L$ are anomalous. Thus, in order to gauge $U(1)_B$ or $U(1)_L$, we need to add new fermions charged under the EW symmetry + $U(1)_B$ or $U(1)_L$.

Also, since their role is to cancel the mixed anomaly, they must be chiral under EW symmetry or $U(1)_B / U(1)_L$.

This also necessitates they get mass from our Higgs doublet or a new Higgs boson, which provides chiral symmetry breaking.

Next lecture: Jan. 11, 2019

Touch on gravitational anomalies, Witten anomaly, scale anomaly.

And QCD chiral symmetry breaking + $\pi^0 \to \gamma\gamma$.