Lecture 7. 

11-30-18. 1 hour lecture.

Brief discussion of Adler-Bardeen thm.

Non-Abelian anomalies

Anomaly cancellation in the Standard Model. (next time)

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Adler-Bardeen thm.

1-loop result for ABJ anomaly is exact. No modification at 2-loop. Anomaly coefficient does not renormalize.
$\Rightarrow$ Justification from calculation using Fujikawa's path integral method.

Consequences:

Anomaly is scale-free. Exhibits IR & UV interpretation (cf. Shifman).

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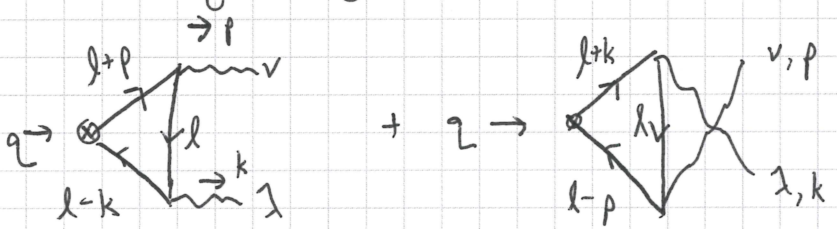
Non-Abelian anomalies.

Recall triangle diagrams for U(1) vector currents & axial current.




$$iM^{\mu\nu\lambda} = (-1)\int \frac{d^4 l}{(2\pi)^4}\,\mathrm{Tr}\left[(i\gamma^\mu\gamma^5)\frac{i(\not l-\not k)}{(l-k)^2}(-ie\gamma^\lambda)\frac{i\not l}{l^2}(-ie\gamma^\nu)\frac{i(\not l+\not p)}{(l+p)^2}\right]$$
$$+\ (p,\nu)\longleftrightarrow(k,\lambda)$$

For non-Abelian gauge couplings + non-Abelian axial vector current, the matrix element now includes an additional group generator at each vertex. Since group symmetry factors separate from the fermion trace, we get

$$iM^{\mu\nu\lambda}_{\text{non-Abelian}} = \mathrm{Tr}\left[t^A\{t^b,t^c\}\right]\cdot iM^{\mu\nu\lambda}_{\substack{\text{pure}\\ \text{Abelian}\\ U(1)}}$$

Then $A$, non-Abelian anomaly coefficient, is related to the familiar Abelian anomaly coefficient as

$$A^{\text{non-Abelian}} = \mathrm{Tr}[\, t^a, \{t^b, t^c\}]\, A^{ABJ}$$

↑ Adler-Bell-Jackiw $U(1)$ anomaly.

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General rules for cancelling anomalies:

Given a set of fermion content:

① If the fermions are not chiral, then they contribute nothing to the anomaly coefficient. In particular, non-chiral fermions admit a tree-level mass term in the Lagrangian, and hence they will not have any irreducible effect in the deep IR.

(Contrapositively, the chiral fermions cannot have a tree-level mass term in the Lagrangian (since it would violate the chiral symmetry) and contribute a nonzero anomaly coefficient.

Hence, vector-like reps. are not anomalous.

② Fermions transforming as real irreps. have no anomaly contrib.

(eg. adjoint in $SU(N)$, all reps. in $SO(N)$.)

For real irreps, $t^a_r = t^a_{\bar{r}}$.

③ $\sum_f A^{\substack{\text{all possible pure + mixed} \\ \text{currents as ext. vectors}}} = 0$ is a consistency condition for an ultraviolet description of a chiral gauge theory.

If violated, gauge symmetry is violated since Ward identity is not conserved.