Lecture 4.

11-8-18. (Thursday.)

Clarify from last time.

γ^5 anti-commutes with γ^μ for $\mu = 0,1,2,3$.
Commutes otherwise.

Identity: $q_\mu \gamma^\mu \gamma^5 = (\not{l} + \not{p} - \not{l} + \not{k})\gamma^5 = (\not{l}+\not{p})\gamma^5 - (\not{l} - \not{k})\gamma^5$

$$= (\not{l}+\not{p})\gamma^5 - (\not{l}_{\parallel} + \not{l}_{\perp} - \not{k})\gamma^5$$

$$= (\not{l}+\not{p})\gamma^5 + \gamma^5(\not{l}_{\parallel} - \not{l}_{\perp} - \not{k})$$

$$= (\not{l}+\not{p})\gamma^5 + \gamma^5(\not{l} - \not{k}) - 2\gamma^5 \not{l}_{\perp}$$

Tips + tricks / discussion for calculating anomalies.

1. Momentum shifts of divergent integrals is, in general, dangerous.

Argument from Schwartz, Sec. 30.2.2.

$$\Delta(a) = \int_{-\infty}^{\infty} dx \; [f(x+a) - f(x)]$$

for $f(x) \underset{x\to+\infty}{\to} c_1$ + $f(x) \underset{x\to-\infty}{\to} c_2$. Each term is linearly divergent.

If we could simply shift, then integrand is 0 + $\Delta(a)$ vanishes.

Instead, Taylor expand.

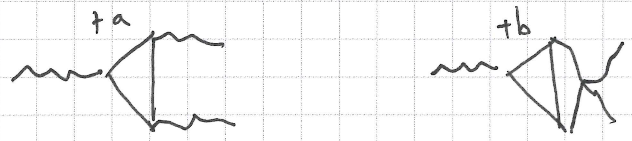
$$\Delta(a) = \int_{-\infty}^{\infty} dx \left[a f'(x) + \frac{a^2}{2} f''(x) + \dots \right] = a\left[f(\infty) - f(-\infty)\right]$$

$$= a(c_1 - c_2).$$ Result is proportional to shift.

In 4D, same argument applies. When evaluating triangle diagrams, the loop momentum is arbitrary, but once it is fixed, the evaluation must be consistent. In particular, dim. reg. forces the loop momentum choice to preserve the Ward identity of vector current, + hence the axial current picks up the anomaly.

On a similar note,

(2) Physics should be invariant wrt calculational choices.



$i\mathcal{M}_{tot} = i\mathcal{M}_1 + i\mathcal{M}_2 \Rightarrow$ Can break apart integrals of $\mathcal{M}_1 + \mathcal{M}_2$ with diff. loop momenta.

Argument from Weinberg with a, b, same as Dedes, Suxho 1202.4940. using w, z.

Vertex function:

$$\Gamma^{\mu\nu\rho} = \left(F_1 p_{1\mu}\epsilon^{\nu\rho\alpha\beta} + F_2 p_{2\mu}\epsilon^{\nu\rho\alpha\beta} + F_3 p_{1\nu}\epsilon^{\rho\mu\alpha\beta} + F_4 p_{2\nu}\epsilon^{\rho\mu\alpha\beta} + F_5 p_{1\rho}\epsilon^{\mu\nu\alpha\beta} + F_6 p_{2\rho}\epsilon^{\mu\nu\alpha\beta}\right) p_{1\alpha}p_{2\beta} + G_1\epsilon^{\mu\nu\rho\alpha} p_{1\alpha} + G_2\epsilon^{\mu\nu\rho\beta} p_{2\beta}$$

Study current conservation at each vertex.

Set $p_1^2 = p_2^2 = 0$.

$$(p_1+p_2)_\mu\Gamma^{\mu\nu\rho} = \left(F_1\, p_1\cdot p_2\, \epsilon^{\nu\rho\alpha\beta} + F_2\, p_1\cdot p_2\,\epsilon^{\nu\rho\alpha\beta}\right)p_{1\alpha}p_{2\beta} + G_1\epsilon^{\mu\nu\rho\alpha}p_{1\alpha}p_{2\mu} + G_2\epsilon^{\mu\nu\rho\beta}p_{1\mu}p_{2\beta}$$

$$(-p_{1\nu})\Gamma^{\mu\nu\rho} = -F_4\, p_1\cdot p_2\,\epsilon^{\rho\mu\alpha\beta}p_{1\alpha}p_{2\beta} - G_2\epsilon^{\mu\nu\rho\beta}p_{1\nu}p_{2\beta}$$

$$(-p_{2\rho})\Gamma^{\mu\nu\rho} = -F_5\, p_1\cdot p_2\,\epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta} - G_1\epsilon^{\mu\nu\rho\alpha}p_{1\alpha}p_{2\rho}$$

If we want gauged current for μ, then need $(p_1+p_2)^2 = 2p_1\cdot p_2 = 0$

[Can allow isolated poles in form factors $F_1 \dots F_6$, so $F\cdot p^2$ gives finite residue.]

Setting $p_1\cdot p_2 = 0$,

$$(p_1+p_2)_\mu\Gamma^{\mu\nu\rho} = -\left(G_1(w,z) + G_2(w,z)\right)\epsilon^{\nu\rho\alpha\beta}p_{1\alpha}p_{2\beta}$$

$$-(p_{1\nu})\Gamma^{\mu\nu\rho} = G_2(w,z)\,\epsilon^{\mu\rho\alpha\beta}p_{1\alpha}p_{2\beta}$$

$$-(p_{2\rho})\Gamma^{\mu\nu\rho} = G_1(w,z)\,\epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}$$

where now we include the shift dependence in G_1, G_2.

Calculation gives

$$(p_1+p_2)_\mu \Gamma^{\mu\nu\rho} = (w-z)\, A\, \epsilon^{\nu\rho\alpha\beta}\, p_{1\alpha} p_{2\beta}$$

$$-p_{1\nu} \Gamma^{\mu\nu\rho} = (w-1)\, A\, \epsilon^{\mu\rho\alpha\beta}\, p_{1\alpha} p_{2\beta}$$

$$-p_{2\rho} \Gamma^{\mu\nu\rho} = (z+1)\, A\, \epsilon^{\mu\nu\alpha\beta}\, p_{1\alpha} p_{2\beta}$$

Hence, we can at most set w, z s.t. two currents are conserved. For the covariant anomaly, with the anomaly A shared among all three currents, we have $w = -z = \frac{1}{3}$.

For the consistent anomaly, with the anomaly only in the μ index, we have $w = -z = 1$. Dim. reg. uses this prescription.

More tips.

3. Cannot reverse traces of Dirac matrices involving γ^5.

$$\mathrm{Tr}[\gamma^\alpha \gamma^\beta \ldots \gamma^\omega \gamma^5] \neq \mathrm{Tr}[\gamma^5 \gamma^\omega \ldots \gamma^\beta \gamma^\alpha]$$

See Körner, Kreimer, Schilcher, Zeitschrift Phys. C. 1992, 54 3, 503.

4. Feynman params. have symmetry in assigning denominators.

$$\frac{1}{d_1 d_2 d_3} = \int \frac{dx\, dy\, dz\; 2\,\delta(x+y+z-1)}{(x d_1 + y d_2 + z d_3)^3}$$

or other combinations of $(x,y,z) \leftrightarrow (d_1, d_2, d_3)$