Mainz Institute for Theoretical Physics

Felix Yu: Chirality and Gauge Theories

Lecture 1.

①

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Motivation for Advanced QFT.

①. Two lecture courses in QFT builds up just enough to get to understand Standard Model.

Tendency to then learn the phenomenology of the SM, but not so much the SM as a QFT.

⇒ Distinction is analogous to Yuval Grossman's distinction between studying The SM vs. A SM.

i.e. All SMs exhibit CKM. Ours is mostly $\mathbb{1}$ + largest off-diagonal is Cabibbo.

All SMs exhibit absence of tree-level FCNCs

All SMs exhibit mass-coupling proportionality, if Higgs vev ≫ SU(2) confining scale.
- open question (lattice?) whether chiral gauge theories break chiral symmetry in their confining phase

Picture builds up that some problems + proposed solutions in BSM physics are similarly generic problems for all SMs or specific problems to our SM.

Example: NP flavor puzzle. ⇒ statement about global symmetry structure pattern in NP interactions with us. No real reason except it works.

Baryogenesis ⇒ Our SM does not predict sufficient/any EW baryogenesis, but diff. SMs could/do.

Hierarchy problem ⇒ Radiative stability of EW scale is generically solved by SUSY.

(2) Learning the "rules" "tools" + for model-building in BSM and how to break the rules.

Tools: symmetries + field content.

If you write random field content and gauge symmetries, theory can be inconsistent.
⇒ chiral anomalies.
⇒ but can fix up by treating as EFT ⇒ what goes wrong + how to fix?

If you adopt particular parameters, theory can have limited / no meaningful range of validity.
⇒ IR or UV Landau poles.
⇒ Descriptions for how to understand IR degrees of freedom. Less / no strong guidance for UV Landau poles.

⇒ Unstable vacua.
⇒ Can be desireable, i.e. phase transitions

In this course, will have 2 main topics:

(1) Chiral anomalies.
Related subtopic: Witten's $SU(2)$ anomaly + scale anomaly.

(2) Topology in field theory, Θ-vacuum of Yang-Mills.
Subtopic: Instantons in Yang-Mills, strong CP
Sphalerons in EW.

Part 1. Chiral anomalies.
aka. Cannot write down random field content for BSM.
aka. Why # of quark generations = # of lepton generations.

Refs. Peskin + Schroeder, Chap. 19.

Bilal, Lectures on Anomalies, 0802.0634

Shifman, Advanced Topics in QFT, Chap. 8

Study 1+1 QED, Schwinger model. Phys. Rev. 128, 2425 (1962)

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Note: derivative always has mass dimension 1.

$$(\partial_\mu \phi)^2 \Rightarrow [\phi] = 0$$

$$i\bar{\Psi}\gamma^\mu\partial_\mu\Psi \Rightarrow [\Psi] = 1/2$$

$$F_{\mu\nu}F^{\mu\nu} \Rightarrow [A_\mu] = 0$$

$$D_\mu = \partial_\mu + ieA_\mu \Rightarrow [e] = 1.$$

Construct Dirac algebra.

Require $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

Use representations $\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

Define $\gamma^5 = \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

For $m=0$, $\mathcal{L}$ is separable + expect conservation of components $\Psi = \begin{pmatrix} \Psi_+ \\ \Psi_- \end{pmatrix}$, labeled by γ^5 eigenvalues.

Can write $\mathcal{L} = \Psi_+^\dagger\, i(D_0 + D_1)\Psi_+ + \Psi_-^\dagger\, i(D_0 - D_1)\Psi_- - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

In free theory, $A_\mu = 0$, field equations of Ψ_+ + Ψ_- are

$$i(\partial_0 + \partial_1)\Psi_+ = 0 \Rightarrow \Psi_+ \propto \phi(x-t) \quad \text{right-moving wave}$$

$$i(\partial_0 - \partial_1)\Psi_- = 0 \Rightarrow \Psi_- \propto \phi(x+t) \quad \text{left-moving wave}$$

Can write number currents, expect conserved

$$N_R \equiv \int dx'\, \Psi_+^\dagger \Psi_+$$

$$N_L \equiv \int dx'\, \Psi_-^\dagger \Psi_-$$

which arise from the spatial integral (cf. conserved charge) of

$$j_R^\mu = \bar{\Psi}\gamma^\mu \frac{1+\gamma^5}{2}\Psi$$

$$j_L^\mu = \bar{\Psi}\gamma^\mu \frac{1-\gamma^5}{2}\Psi.$$

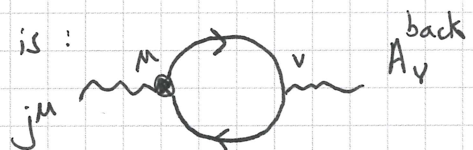
Expectation still true in interacting theory, since $\mathcal{L}$ is still separable. As a consequence, expect $j^\mu = j^\mu_R + j^\mu_L$ & $j^{\mu 5} = j^\mu_R - j^\mu_L$ are both conserved.

In 2D, vector + axial-vector currents are not independent. Have identity $\gamma^\mu \gamma^5 = -\epsilon^{\mu\nu}\gamma_\nu$

Will calculate $\partial_\mu j^{\mu 5}$, check if 0.

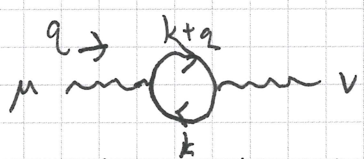
First method, calculate exp. value $\langle j^\mu \rangle$ by coupling j^μ to background electric field. Later replace $\langle j^\mu \rangle$ by $\langle j^{\mu 5} \rangle$.

Diagram is:



Middle structure is the amputated 2-pt. correlation function for two photons, aka vacuum polarization fcn.

Review: Adopt dim. reg. $\Leftarrow$ Important! Regularization preserves gauge invariance.



One-loop order, scales as e^2.

$$i\Pi_2^{\mu\nu}(q) = (-1)(-ie)^2 \int \frac{d^dk}{(2\pi)^d} \mathrm{Tr}\left[\gamma^\mu \frac{i(\not{k}+m)}{k^2-m^2}\gamma^\nu \frac{i(\not{k}+\not{q}+m)}{(k+q)^2-m^2}\right]$$

$$= \frac{-e^2}{(2\pi)^d}\int d^dk \, \mathrm{Tr}\left[\frac{\gamma^\mu(\not{k}+m)\gamma^\nu(\not{k}+\not{q}+m)}{(k^2-m^2)\left((k+q)^2-m^2\right)}\right]$$

Feynman parameters: $\displaystyle \frac{1}{AB} = \int_0^1 dx \frac{1}{(xA+(1-x)B)^2}$

...

$$= -\frac{e^2}{(2\pi)^d}\int d^dk \int dx \frac{\mathrm{Tr}\left[\gamma^\mu(\not{k}+m)\gamma^\nu(\not{k}+\not{q}+m)\right]}{((k+(1-x)q)^2-\Delta)^2}$$

For $\Delta \equiv (1-x)^2 q^2 - (1-x) q^2 + m^2$

Shift $l \equiv k + (1-x) q$

$$= \frac{-e^2}{(2\pi)^d} \int d^d l \int dx \; \mathrm{Tr}\left[\frac{\gamma^\mu (\not{l} - (1-x)\not{q} + m) \gamma^\nu (\not{l} + x\not{q} + m)}{(l^2-\Delta)^2} \right]$$

Solve trace:

$$\mathrm{Tr}[\ldots] = \mathrm{Tr}[1] \left(2 l^\mu l^\nu - g^{\mu\nu} l^2 + -(1-x)x \left(2 q^\mu q^\nu - q^2 g^{\mu\nu}\right) + m^2 g^{\mu\nu} \right)$$

Look up:

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^\mu l^\nu}{(l^2-\Delta)^2} = \frac{(-1)\, i}{(4\pi)^{d/2}} \frac{g^{\mu\nu}}{2} \frac{\Gamma(1-\frac{d}{2})}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}}$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l^2}{(l^2-\Delta)^2} = \frac{(-1)\, i}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma(1-\frac{d}{2})}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}}$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l^2-\Delta)^2} = \frac{(-1)^2\, i}{(4\pi)^{d/2}} \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}}$$

Leading divergence:

$$= \frac{-e^2}{(2\pi)^d} \int d^d l \int dx \; \mathrm{Tr}[1] \frac{\left(2 l^\mu l^\nu - g^{\mu\nu} l^2\right)}{(l^2-\Delta)^2}$$

$$= \left(-e^2 \int dx \; \mathrm{Tr}[1]\right) \left(\frac{(-1)\, i}{(4\pi)^{d/2}} \frac{1}{2} \frac{\Gamma(1-\frac{d}{2})}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{1-\frac{d}{2}} \right) \left(2 g^{\mu\nu} - d g^{\mu\nu}\right)$$

$$= \left(-e^2 \int dx \; \mathrm{Tr}[1]\right) \left(\frac{-i}{(4\pi)^{d/2}} \cdot \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} \cdot \Delta g^{\mu\nu} \right)$$

Combine

$$= \left(-e^2 \int dx \; \mathrm{Tr}[1]\right) \left(\frac{i}{(4\pi)^{d/2}} \frac{\Gamma(2-\frac{d}{2})}{\Gamma(2)} \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}} \right)$$
$$\left(-\Delta g^{\mu\nu} - x(1-x) \left(2 q^\mu q^\nu - q^2 g^{\mu\nu}\right) + m^2 g^{\mu\nu} \right)$$

$\Delta = -x(1-x) q^2 + m^2$, set $d = 2$.

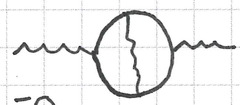
$$= \left(-e^2 \int dx \; \mathrm{Tr}[1]\right) \left(\frac{i}{4\pi} \frac{1}{-x(1-x) q^2 + m^2} \right)$$
$$\cdot \left(2 \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\right) \left(-x(1-x)\right)$$

Take $m=0$

$$= \left(-e^2 \int dx \; \mathrm{Tr}[\mathbb{1}]\right) \frac{i}{2\pi} \cdot \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)$$

$$i\Pi_2^{\mu\nu}(q) = \frac{ie^2}{\pi}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \qquad m_\gamma^2 = \frac{e^2}{\pi}$$

Establish by propagator calc.
Note higher order diagrams vanish.



$$\gamma_\mu \gamma^\nu \gamma^\mu = 2g^{\mu\nu}\gamma_\mu - \gamma^\nu\gamma^\mu\gamma_\mu = 0$$

Full photon propagator: dressed

$$i\Pi^{\mu\nu}(q^2) = \frac{-ig^{\mu\nu}}{q^2} + \frac{-ig^{\mu\rho}}{q^2}\left(\frac{ie^2}{\pi}\left(g^{\rho\sigma} - \frac{q^\rho q^\sigma}{q^2}\right)\right)\frac{-ig^{\sigma\nu}}{q^2}$$

$$+ \frac{-ig^{\mu\alpha}}{q^2}\left(\frac{ie^2}{\pi}\left(g^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2}\right)\right)\frac{-ig^{\beta\gamma}}{q^2}$$

$$\times \left(\frac{ie^2}{\pi}\left(g^{\gamma\delta} - \frac{q^\gamma q^\delta}{q^2}\right)\right)\frac{-ig^{\delta\nu}}{q^2} + \ldots$$

$$= \frac{-ig^{\mu\nu}}{q^2} - i\frac{e^2}{q^4}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) - \frac{ie^4}{q^6\pi^2}\left(g^{\mu\beta} - \frac{q^\mu q^\beta}{q^2}\right)\left(g^{\beta\nu} - \frac{q^\beta q^\nu}{q^2}\right) + \ldots$$

$$= \frac{-ig^{\mu\nu}}{q^2} - i\frac{e^2}{\pi q^4}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) - \frac{ie^4}{q^6\pi^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} - \cancel{\frac{q^\mu q^\nu}{q^2}} + \cancel{\frac{q^\mu q^\nu}{q^2}}\right) + \ldots$$

$$= \frac{-ig^{\mu\nu}}{q^2} - \frac{i}{q^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\left(\frac{e^2}{\pi q^2} + \frac{e^4}{\pi^2 q^4} + \ldots\right)$$

$$= -\frac{ig^{\mu\nu}}{q^2} - \frac{i}{q^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\left(\frac{1}{1 - \frac{e^2}{\pi q^2}}\right)$$

$$= \frac{-ig^{\mu\nu}}{q^2} - i\frac{1}{q^2 - \frac{e^2}{\pi}}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \Rightarrow \text{mass} = \sqrt{\frac{e^2}{\pi}}$$

transverse $\qquad$ longitudinal